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一般退化中立型微分系统解的存在性及通解

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摘　要:讨论了对于退化矩阵 $\boldsymbol{E}$ 不是方阵情形的一般退化中立型微分系统的解，基于退化的常微分系统解的存在性条件，通过定义可解阵对和基础解以及利用拉普拉斯变换，给出了一般退化中立型微分系统解的存在性条件以及通解表达式。

关键词:退化中立型微分方程；可解阵对；存在性；通解

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Existence of the solution for the general degenerate neutral differential system and the general solution

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Abstract: This paper deals with the solution of the general degenerate neutral differential system when the degenerate matrix $\boldsymbol{E}$ is not square. Based on the existence condition of the solution for singular ordinary differential systems，the existence condition of the solution of the degenerate neutral differential system as well as the general solution are obtained by defining the solvable matrix pair and fundamental solution and using Laplace transformation.

Key words: degenerate neutral differential equation；solvable matrix pair；existence；general solution

近年来，关于退化时滞微分系统的研究已经成为系统理论和应用学科的新课题[1-10]。本文研究了下述较一般的退化中立型微分系统相容性条件以及通解问题

$$\begin{cases}\boldsymbol{E}\dot{x}(t)=\boldsymbol{A}x(t)+\boldsymbol{B}x(t-1)+\\ \qquad\boldsymbol{C}\dot{x}(t-1)+\boldsymbol{f}(t)\quad t\geqslant 0\\ x(t)=\varphi(t)\qquad -1\leqslant t\leqslant 0\end{cases}\tag{1}$$

其中 $x(t)\in R^n$，$\boldsymbol{E},\boldsymbol{A},\boldsymbol{B},\boldsymbol{C}\in R^{m\times n}$，$\boldsymbol{f}(t)\in R^m$，$\mathrm{rank}(\boldsymbol{E})<n$。

这里总假定 $\boldsymbol{A}$ 的列向量线性无关。

对于系统(1)的研究，一般考虑的是 $m=n$ 情形，文献[1]给出了在 $m=n$ 情形下的常数变易公式和通解表达式，而对 $m\neq n$ 的情形研究得比较少。本文首先通过定义可解阵对来讨论系统(1)解的存在惟一性，然后通过定义基础解以及利用 Laplace 变换，给出系统(1)的通解表达式。

1　预备知识

定义 1　对于任何方阵 $\boldsymbol{P}\in R^{n\times n}$，如果存在一个矩阵 $\boldsymbol{P}^d$ 满足：① $\boldsymbol{P}\boldsymbol{P}^d=\boldsymbol{P}^d\boldsymbol{P}$；② $\boldsymbol{P}^d\boldsymbol{P}\boldsymbol{P}^d=\boldsymbol{P}^d$；③ $(\boldsymbol{I}_n-\boldsymbol{P}^d\boldsymbol{P})\boldsymbol{P}^l=0$。

则称 $\boldsymbol{P}^d$ 为矩阵 $\boldsymbol{P}$ 的 Drazin 逆，其中 l 是矩阵 $\boldsymbol{P}$ 的指标，亦即使得 $\mathrm{rank}(\boldsymbol{P}^{l+1})=\mathrm{rank}(\boldsymbol{P}^l)$ 成立的最小非负整数，记作 $\mathrm{ind}(\boldsymbol{P})=l$。

定义 2　对于矩阵 $\boldsymbol{E},\boldsymbol{A}\in R^{m\times n}$，若 $\boldsymbol{A}$ 的列向量线性无关。定义 $\boldsymbol{E}^A=(\boldsymbol{Y}\boldsymbol{E})^d\boldsymbol{Y}$，$\boldsymbol{Y}\in R^{n\times m}$满足

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$$(\boldsymbol{I}_m - \boldsymbol{AY})(\boldsymbol{EY})^d = 0, \quad \boldsymbol{YA} = \boldsymbol{I}_n \tag{2}$$

则称矩阵对($\boldsymbol{E}^A,\boldsymbol{Y}$)为($\boldsymbol{E},\boldsymbol{A}$)的可解阵对。

引理 1[4]　① $\boldsymbol{E}^A = \boldsymbol{E}^A\boldsymbol{E}\boldsymbol{E}^A$；② $(\boldsymbol{E}^A\boldsymbol{E})^2 = \boldsymbol{E}^A\boldsymbol{E}$；③ $\boldsymbol{AE}^A\boldsymbol{E} = \boldsymbol{EE}^A\boldsymbol{A}$；④ $\boldsymbol{E}^A\boldsymbol{AY} = \boldsymbol{E}^A$；⑤ $\boldsymbol{E}^A\boldsymbol{AE}^A\boldsymbol{E} = \boldsymbol{E}^A\boldsymbol{EE}^A\boldsymbol{A} = \boldsymbol{E}^A\boldsymbol{A}$；⑥ $\boldsymbol{AYEE}^A = \boldsymbol{EE}^A\boldsymbol{AY} = \boldsymbol{EE}^A$；⑦ $\boldsymbol{E}^A\boldsymbol{EYE} = \boldsymbol{YEE}^A\boldsymbol{E}$；⑧ $[(\boldsymbol{I}_n - \boldsymbol{E}^A\boldsymbol{E})\boldsymbol{YE}]^h = (\boldsymbol{I}_n - \boldsymbol{E}^A\boldsymbol{E})(\boldsymbol{YE})^h \quad (h = \mathrm{ind}(\boldsymbol{YE}))$。

考虑退化的常微系统

$$\begin{cases} \boldsymbol{E}\dot{x}(t) = \boldsymbol{A}x(t) + \boldsymbol{f}(t) & t \geqslant 0 \\ x(0) = x_0 \end{cases} \tag{3}$$

其中，$x(t) \in R^n$，$\boldsymbol{E}, \boldsymbol{A} \in R^{m\times n}$，$f(t) \in R^m$，$\mathrm{rank}(\boldsymbol{E}) < n$，假定 A 的列向量线性无关。

引理 2[6]　对于系统(3)，($\boldsymbol{E}^A,\boldsymbol{Y}$)为($\boldsymbol{E},\boldsymbol{A}$)的可解阵对，$\mathrm{ind}(\boldsymbol{YE}) = h$，假定 $\boldsymbol{f}(t)$ 具有 h 阶导数，则系统(3)存在惟一解的充分必要条件是下列 2 式成立

$$(\boldsymbol{I}_m - \boldsymbol{AY})\sum_{i=0}^{h}(\boldsymbol{EY})^i\boldsymbol{f}^{(i)}(t) = 0 \quad (t \geqslant 0) \tag{4}$$

$$(\boldsymbol{I}_n - \boldsymbol{E}^A\boldsymbol{E})x_0 = -(\boldsymbol{I}_n - \boldsymbol{E}^A\boldsymbol{E})\sum_{i=0}^{h-1}(\boldsymbol{YE})^i\boldsymbol{Y}\boldsymbol{f}^{(i)}(0) \tag{5}$$

并且在此充要条件成立时，系统(3)的解为

$$x(t) = \mathrm{e}^{\boldsymbol{E}^A\boldsymbol{A}t}\boldsymbol{E}^A\boldsymbol{E}x_0 + \int_0^t \mathrm{e}^{\boldsymbol{E}^A\boldsymbol{A}(t-\tau)}\boldsymbol{E}^A\boldsymbol{f}(\tau)\mathrm{d}\tau - (\boldsymbol{I}_n - \boldsymbol{E}^A\boldsymbol{E})\sum_{i=0}^{h-1}(\boldsymbol{YE})^i\boldsymbol{Y}\boldsymbol{f}^{(i)}(t) \quad (t \geqslant 0) \tag{6}$$

2　主要结果

先分析系统(1)的可解性。

定理 1　对于系统(1)，($\boldsymbol{E}^A,\boldsymbol{Y}$)为($\boldsymbol{E},\boldsymbol{A}$)的可解阵对，$\mathrm{ind}(\boldsymbol{YE}) = h$，假定 $\boldsymbol{f}(t)$，$\boldsymbol{\varphi}(t)$具有 $h+1$ 阶导数，并且$(\boldsymbol{I}_m - \boldsymbol{AY})(\boldsymbol{EY})^i\boldsymbol{B} = 0$，$(\boldsymbol{I}_m - \boldsymbol{AY})(\boldsymbol{EY})^i\boldsymbol{C} = 0$，$(i = 0,1,2,\cdots,h)$，则系统(1)存在惟一解的充分必要条件是下列 2 式成立

$$(\boldsymbol{I}_m - \boldsymbol{AY})\sum_{i=0}^{h}(\boldsymbol{EY})^i\boldsymbol{f}^{(i)}(t) = 0 \quad (t \geqslant 0) \tag{7}$$

$$(\boldsymbol{I}_n - \boldsymbol{E}^A\boldsymbol{E})\varphi(0) = -(\boldsymbol{I}_n - \boldsymbol{E}^A\boldsymbol{E})\sum_{i=0}^{h-1}(\boldsymbol{YE})^i\boldsymbol{Y}[\boldsymbol{B\varphi}^{(i)}(-1) + \boldsymbol{C\varphi}^{(i+1)}(-1) + f^{(i)}(0)] \tag{8}$$

证明

(1) 充分性。利用分步法分析。当 $t \in [0,1]$时，系统(1)变为

$$\begin{cases} \boldsymbol{E}\dot{x}(t) = \boldsymbol{A}x(t) + \boldsymbol{B\varphi}(t-1) + \boldsymbol{C\dot{\varphi}}(t-1) + \boldsymbol{f}(t) & 0 \leqslant t \leqslant 1 \\ \boldsymbol{x}(0) = \boldsymbol{\varphi}(0) \end{cases}$$

此为退化的常微系统。由假定的条件及(7)式，可得

$$(\boldsymbol{I}_m - \boldsymbol{AY})\sum_{i=0}^{h}(\boldsymbol{EY})^i[\boldsymbol{B\varphi}^{(i)}(t-1) + \boldsymbol{C\varphi}^{(i+1)}(t-1) + f^{(i)}(t)] = 0$$

再由(8)式并根据引理 2 知，系统(1)在 $t \in [0,1]$上存在惟一解，且其解为

$$x(t) = \mathrm{e}^{\boldsymbol{E}^A\boldsymbol{A}t}\boldsymbol{E}^A\boldsymbol{E}\varphi(0) + \int_0^t \mathrm{e}^{\boldsymbol{E}^A\boldsymbol{A}(t-\tau)}\boldsymbol{E}^A[\boldsymbol{B\varphi}(\tau-1) + \boldsymbol{C\dot{\varphi}}(\tau-1) + f(\tau)]\mathrm{d}\tau - (\boldsymbol{I}_n - \boldsymbol{E}^A\boldsymbol{E})\sum_{i=0}^{h-1}(\boldsymbol{YE})^i\boldsymbol{Y}[\boldsymbol{B\varphi}^{(i)}(t-1) + \boldsymbol{C\varphi}^{(i+1)}(t-1) + f^{(i)}(t)] \quad (t \in [0,1]) \tag{9}$$

类似分析，当 $t \in [1,2]$时，系统(1)的解也存在惟一，系统(1)在$[0,+\infty)$上存在惟一解。

(2) 必要性。如果系统(1)在$[0,+\infty)$上存在惟一解，则系统(1)在$[0,1]$上其解也存在惟一，故由引理 2 并结合假定条件知，(8)式成立，(7)式在$[0,1]$上成立。同样，在$[1,2]$上系统(1)式的解也存在惟一，从而(7)式在$[1,2]$上也成立。由归纳法知，(7)式在$[0,+\infty)$上均成立，证毕。

定理 2　设系统(1)满足定理 1 的条件，则系统(1)同解于下述系统

$$\begin{cases} \dot{x}(t) = \boldsymbol{E}^A[\boldsymbol{A}x(t) + \boldsymbol{B}x(t-1) + \boldsymbol{C}\dot{x}(t-1) + f(t)] - \\ \quad (\boldsymbol{I}_n - \boldsymbol{E}^A\boldsymbol{E})\sum_{i=0}^{h-1}(\boldsymbol{YE})^i\boldsymbol{Y}[\boldsymbol{B}x^{(i+1)}(t-1) + \boldsymbol{C}x^{(i+2)}(t-1) + f^{(i+1)}(t)] & t \geqslant 0 \\ x(t) = \varphi(t) & -1 \leqslant t \leqslant 0 \end{cases} \tag{10}$$

证明　在系统(1)的两边分别左乘 $\boldsymbol{E}^A$ 及$(\boldsymbol{I}_n - \boldsymbol{E}^A\boldsymbol{E})\boldsymbol{Y}$，可得

$$\boldsymbol{E}^A\boldsymbol{E}\dot{x}(t) = \boldsymbol{E}^A\boldsymbol{A}x(t) + \boldsymbol{E}^A\boldsymbol{B}x(t-1) + \boldsymbol{E}^A\boldsymbol{C}\dot{x}(t-1) + \boldsymbol{E}^Af(t) \tag{11}$$

$$(\boldsymbol{I}_n-\boldsymbol{E}^A\boldsymbol{E})\boldsymbol{Y}\boldsymbol{E}\dot{x}(t)=(\boldsymbol{I}_n-\boldsymbol{E}^A\boldsymbol{E})\boldsymbol{Y}[\boldsymbol{A}x(t)+\boldsymbol{B}x(t-1)+\boldsymbol{C}\dot{x}(t-1)+f(t)] \tag{12}$$

利用引理1的(2)式和(7)式,可把(12)式变为

$$(\boldsymbol{I}_n-\boldsymbol{E}^A\boldsymbol{E})\boldsymbol{Y}\boldsymbol{E}[(\boldsymbol{I}_n-\boldsymbol{E}^A\boldsymbol{E})\dot{x}(t)]=(\boldsymbol{I}_n-\boldsymbol{E}^A\boldsymbol{E})\boldsymbol{Y}[\boldsymbol{A}x(t)+\boldsymbol{B}x(t-1)+\boldsymbol{C}\dot{x}(t-1)+f(t)] \tag{13}$$

注意到$\boldsymbol{YA}=\boldsymbol{I}_n$,$\mathrm{ind}(\boldsymbol{YE})=h$,把整个$(\boldsymbol{I}_n-\boldsymbol{E}^A\boldsymbol{E})x(t)$当作未知函数,迭代(13)式$h-1$次,可得

$$(\boldsymbol{I}_n-\boldsymbol{E}^A\boldsymbol{E})x(t)=-\sum_{i=0}^{h-1}(\boldsymbol{I}_n-\boldsymbol{E}^A\boldsymbol{E})(\boldsymbol{YE})^i\boldsymbol{Y}[\boldsymbol{B}x^{(i)}(t-1)+\boldsymbol{C}x^{(i+1)}(t-1)+f^{(i)}(t)]$$

$$(\boldsymbol{I}_n-\boldsymbol{E}^A\boldsymbol{E})\dot{x}(t)=-\sum_{i=0}^{h-1}(\boldsymbol{I}_n-\boldsymbol{E}^A\boldsymbol{E})(\boldsymbol{YE})^i\boldsymbol{Y}[\boldsymbol{B}x^{(i+1)}(t-1)+\boldsymbol{C}x^{(i+2)}(t-1)+f^{(i+1)}(t)]$$

把(11)式与上式相加,可得

$$\begin{aligned}\dot{x}(t)=\ &\boldsymbol{E}^A[\boldsymbol{A}x(t)+\boldsymbol{B}x(t-1)+\boldsymbol{C}\dot{x}(t-1)+f(t)]-\\&(\boldsymbol{I}_n-\boldsymbol{E}^A\boldsymbol{E})\sum_{i=0}^{h-1}(\boldsymbol{YE})^i\boldsymbol{Y}[\boldsymbol{B}x^{(i+1)}(t-1)+\boldsymbol{C}x^{(i+2)}(t-1)+f^{(i+1)}(t)]\end{aligned} \tag{14}$$

从而在满足定理1的条件下,系统(1)与系统(10)同解,证毕。

下面定义基础解,利用Laplace变换给出系统(10)即系统(1)的通解表达式。

定义3 若$\boldsymbol{X}(t)$满足

$$\begin{cases}\dot{\boldsymbol{X}}(t)=\boldsymbol{E}^A[\boldsymbol{A}\boldsymbol{X}(t)+\boldsymbol{B}\boldsymbol{X}(t-1)+\boldsymbol{C}\dot{\boldsymbol{X}}(t-1)]-\\\qquad(\boldsymbol{I}_n-\boldsymbol{E}^A\boldsymbol{E})\sum_{i=0}^{h-1}(\boldsymbol{YE})^i\boldsymbol{Y}[\boldsymbol{B}x^{(i+1)}(t-1)+\boldsymbol{C}x^{(i+2)}(t-1)]\\\boldsymbol{X}(t)=\begin{cases}\boldsymbol{I}_n & t=0\\0 & -1\leqslant t<0\end{cases}\end{cases} \tag{15}$$

则称$\boldsymbol{X}(t)$为系统(10)的基础解。

定理3 若$\boldsymbol{X}(t)$为系统(10)的基础解,且$\boldsymbol{YBYE}=\boldsymbol{YEYB}$,$\boldsymbol{YCYE}=\boldsymbol{YEYC}$,则

$$\boldsymbol{X}(t)=L^{-1}[\boldsymbol{H}^{-1}(\lambda)] \tag{16}$$

其中,$\boldsymbol{H}(\lambda)=\lambda\boldsymbol{I}_n-\boldsymbol{E}^A\boldsymbol{A}-\mathrm{e}^{-\lambda}\boldsymbol{E}^A\boldsymbol{B}-\lambda\mathrm{e}^{-\lambda}\boldsymbol{E}^A\boldsymbol{C}+\mathrm{e}^{-\lambda}\sum_{i=0}^{h-1}\lambda^{i+1}(\boldsymbol{I}_n-\boldsymbol{E}^A\boldsymbol{E})(\boldsymbol{YE})^i\boldsymbol{Y}(\boldsymbol{B}+\lambda\boldsymbol{C})$。

证明 在(15)式两边实行Laplace变换,可得$\boldsymbol{H}(\lambda)L[\boldsymbol{X}(t)]=\boldsymbol{X}(0)=\boldsymbol{I}_n$,从而

$$\boldsymbol{X}(t)=L^{-1}[\boldsymbol{H}^{-1}(\lambda)] \tag{17}$$

定理4 设系统(1)满足定理1的条件,且$\boldsymbol{YBYE}=\boldsymbol{YEYB}$,$\boldsymbol{YCYE}=\boldsymbol{YEYC}$,则系统(1)的通解为

$$\begin{aligned}x(t)=\ &\boldsymbol{X}(t)\varphi(0)+\int_0^t\boldsymbol{X}(t-\theta)\boldsymbol{E}^A\boldsymbol{B}\varphi(\theta-1)U(\theta)\mathrm{d}\theta+\int_0^t\boldsymbol{X}(t-\theta)\boldsymbol{E}^Af(\theta)\mathrm{d}\theta+\\&\int_0^t\boldsymbol{X}(t-\theta)\boldsymbol{E}^A\boldsymbol{C}\dot{\omega}(\theta-1)\mathrm{d}\theta-\int_0^t\dot{\boldsymbol{X}}(t-\theta-1)\boldsymbol{E}^A\boldsymbol{C}\omega(\theta)\mathrm{d}\theta-\\&\sum_{i=0}^{h-1}(\boldsymbol{I}_n-\boldsymbol{E}^A\boldsymbol{E})(\boldsymbol{YE})^i\boldsymbol{Y}\int_0^t\boldsymbol{X}(t-\theta)[\boldsymbol{B}\omega^{(i+1)}(\theta-1)+\boldsymbol{C}\omega^{(i+2)}(\theta-1)+f^{(i+1)}(\theta)]\mathrm{d}\theta+\\&\sum_{i=0}^{h-1}(\boldsymbol{I}_n-\boldsymbol{E}^A\boldsymbol{E})(\boldsymbol{YE})^i\boldsymbol{YB}\int_0^t\boldsymbol{X}^{(i+1)}(t-\theta-1)\omega(\theta)\mathrm{d}\theta+\\&\sum_{i=0}^{h-1}(\boldsymbol{I}_n-\boldsymbol{E}^A\boldsymbol{E})(\boldsymbol{YE})^i\boldsymbol{YC}\int_0^t\boldsymbol{X}^{(i+2)}(t-\theta-1)\omega(\theta)\mathrm{d}\theta\end{aligned}$$

其中,$\boldsymbol{X}(t)$为系统(10)的基础解,$U(\theta)=\begin{cases}0 & \theta>1\\1 & 0\leqslant\theta\leqslant1\end{cases}$,$\omega(t)=\begin{cases}\sum_{i=0}^{h}\dfrac{\varphi^{(i)}(0)}{i!}t^i & t>0\\\varphi(t) & -1\leqslant t\leqslant0\end{cases}$。

证明 在(10)式两边实行Laplace变换,并把φ代入,得

$$\begin{aligned}\lambda L[x(t)]-\varphi(0)=\ &\boldsymbol{E}^A\boldsymbol{A}L[x(t)]+\mathrm{e}^{-\lambda}\boldsymbol{E}^A\boldsymbol{B}L[x(t)]+\boldsymbol{E}^A\boldsymbol{B}\int_0^1\varphi(t-1)\mathrm{e}^{-\lambda t}\mathrm{d}t+\boldsymbol{E}^AL[f(t)]+\\&\lambda\mathrm{e}^{-\lambda}\boldsymbol{E}^A\boldsymbol{C}L[x(t)]+\boldsymbol{E}^A\boldsymbol{C}L[\dot{\varepsilon}(t-1)]-\lambda\mathrm{e}^{-\lambda}\boldsymbol{E}^A\boldsymbol{C}L[\omega(t)]-\\&(\boldsymbol{I}_n-\boldsymbol{E}^A\boldsymbol{E})\sum_{i=0}^{h-1}(\boldsymbol{YE})^i\boldsymbol{Y}\{\boldsymbol{B}L[x^{(i+1)}(t-1)]+\boldsymbol{C}L[x^{(i+2)}(t-1)]+L[f^{(i+1)}(t)]\}\end{aligned} \tag{18}$$

由于

$$L[x^{(i+1)}(t-1)]=\lambda^{i+1}\mathrm{e}^{-\lambda}L[x(t)]+L[\omega^{(i+1)}(t-1)]-\lambda^{i+1}\mathrm{e}^{-\lambda}L[\omega(t)] \tag{19}$$

将(19)式代入(18)式，经整理可得

$$\begin{aligned}\boldsymbol{H}(\lambda)L[x(t)]=&\varphi(0)+\boldsymbol{E}^A\boldsymbol{B}\int_0^1\varphi(t-1)\mathrm{e}^{-\lambda t}\mathrm{d}t+\boldsymbol{E}^AL[f(t)]+\boldsymbol{E}^A\boldsymbol{C}L[\omega(t-1)]-\\ &\lambda\mathrm{e}^{-\lambda}\boldsymbol{E}^A\boldsymbol{C}L[\omega(t)]-(I_n-\boldsymbol{E}^A\boldsymbol{E})\sum_{i=0}^{h-1}(\boldsymbol{YE})^i\boldsymbol{Y}\{\boldsymbol{B}L[\omega^{(i+1)}(t-1)]+\boldsymbol{C}L[\omega^{(i+2)}(t-1)]+\\ &L[f^{(i+1)}(t)]\}+(I_n-\boldsymbol{E}^A\boldsymbol{E})\sum_{i=0}^{h-1}(\boldsymbol{YE})^i\boldsymbol{Y}\{\boldsymbol{B}\lambda^{i+1}\mathrm{e}^{-\lambda}L[\omega(t)]+\boldsymbol{C}\lambda^{i+2}\mathrm{e}^{-\lambda}L[\omega(t)]\}\end{aligned}$$

于是

$$\begin{aligned}L[x(t)]=&\boldsymbol{H}^{-1}(\lambda)\varphi(0)+\boldsymbol{H}^{-1}(\lambda)\boldsymbol{E}^A\boldsymbol{B}\int_0^1\varphi(t-1)\mathrm{e}^{-\lambda t}\mathrm{d}t+\boldsymbol{H}^{-1}(\lambda)\boldsymbol{E}^AL[f(t)]+\\ &\boldsymbol{H}^{-1}(\lambda)\boldsymbol{E}^A\boldsymbol{C}L[\dot{\omega}(t-1)]-\boldsymbol{E}^A\boldsymbol{C}L[\dot{\boldsymbol{X}}(t-1)]\cdot L[\omega(t)]-\\ &(\boldsymbol{I}_n-\boldsymbol{E}^A\boldsymbol{E})\sum_{i=0}^{h-1}(\boldsymbol{YE})^i\boldsymbol{Y}L[X(t)]\cdot\{\boldsymbol{B}L[\omega^{(i+1)}(t-1)]+\boldsymbol{C}L[\omega^{(i+2)}(t-1)]+L[f^{(i+1)}(t)]\}+\\ &(\boldsymbol{I}_n-\boldsymbol{E}^A\boldsymbol{E})\sum_{i=0}^{h-1}(\boldsymbol{YE})^i\boldsymbol{Y}\{\boldsymbol{B}L[\boldsymbol{X}^{(i+1)}(t-1)]\cdot L[\omega(t)]+\boldsymbol{C}L[\boldsymbol{X}^{(i+2)}(t-1)]\cdot L[\omega(t)]\}\end{aligned}$$

对上式两边取 Laplace 逆变换，即有

$$\begin{aligned}x(t)=&\boldsymbol{X}(t)\varphi(0)+\int_0^t\boldsymbol{X}(t-\theta)\boldsymbol{E}^A\boldsymbol{B}\varphi(\theta-1)U(\theta)\mathrm{d}\theta+\int_0^t\boldsymbol{X}(t-\theta)\boldsymbol{E}^Af(\theta)\mathrm{d}\theta+\\ &\int_0^t\boldsymbol{X}(t-\theta)\boldsymbol{E}^A\boldsymbol{C}\dot{\omega}(\theta-1)\mathrm{d}\theta-\int_0^t\dot{\boldsymbol{X}}(t-\theta-1)\boldsymbol{E}^A\boldsymbol{C}\omega(\theta)\mathrm{d}\theta-\\ &\sum_{i=0}^{h-1}(\boldsymbol{I}_n-\boldsymbol{E}^A\boldsymbol{E})(\boldsymbol{YE})^i\boldsymbol{Y}\int_0^t\boldsymbol{X}(t-\theta)[\boldsymbol{B}\omega^{(i+1)}(\theta-1)+\boldsymbol{C}\omega^{(i+2)}(\theta-1)+f^{(i+1)}(\theta)]\mathrm{d}\theta+\\ &\sum_{i=0}^{h-1}(\boldsymbol{I}_n-\boldsymbol{E}^A\boldsymbol{E})(\boldsymbol{YE})^i\boldsymbol{YB}\int_0^t\boldsymbol{X}^{(i+1)}(t-\theta-1)\omega(\theta)\mathrm{d}\theta+\\ &\sum_{i=0}^{h-1}(\boldsymbol{I}_n-\boldsymbol{E}^A\boldsymbol{E})(\boldsymbol{YE})^i\boldsymbol{YC}\int_0^t\boldsymbol{X}^{(i+2)}(t-\theta-1)\omega(\theta)\mathrm{d}\theta\end{aligned} \tag{20}$$

这即为系统(1)的通解，证毕。

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