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Gas displacing viscous shear thinning liquids from tubes: Effects of cooling before gas injection

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Abstract

The thickness of the layer left behind when gas displaces a temperature-dependent power law liquid from a tube, after a period of static cooling of the liquid to the tube wall, is evaluated in finite element simulations. On the basis of some 300 numerical simulations, the dimensionless layer thickness is correlated with dimensionless cooling time, Pearson number, Biot number and power law index in convenient formulae which will be of value in estimating the effects of a gas delay in gas assisted injection moulding.

Keywords: Heat transfer; Non-Newtonian fluids; Numerical analysis; Polymer processing; Gas assisted moulding

1. Introduction

In a recent paper, (Polynkin et al., 2004), we studied the displacement of a shear thinning liquid from a cylindrical tube by a gas under isothermal conditions. Gravity and surface tension effects were neglected, and the results therefore apply in the limit of high capillary number. These conditions are relevant to gas-assisted injection moulding of plastics (GAIM), which is the application of particular interest to us. An indication of the relevance to other technologies and a review of work in this area are provided in the cited paper. In GAIM gas is injected into the mould under pressure to displace the melt and core out thicker sections of the moulding. Mould filling is often rapid, with little opportunity for the molten polymer to cool, and an isothermal assumption is valid. The results already obtained will apply in this case. In some applications, however, a delay is used before gas injection, allowing the polymer to cool to the mould walls, resulting in an increase in viscosity and the formation of a thicker layer of plastic against the wall. Existing numerical simulations, as reviewed in Polynkin et al. (2002, 2005), for example, capture this effect; however, available results apply to specific cases and no quantitative generalizations have been published. In the present note we provide dimensionless correlations for predicting the developed layer thickness formed when gas displaces a shear thinning liquid from a tube after a period of static cooling (the developed layer thickness is attained within about one diameter of the inlet under the low Reynolds number conditions considered). These results further extend the information available for predicting the outcomes of GAIM. They will be helpful in improving existing computer simulations that simplify the geometry of moulded parts, and represent the channels cored out by the gas as equivalent cylindrical tubes (see for example Moldflow MPI/Gas www.moldflow.com and all other commercial packages). The work is complementary to the development of non-isothermal three-dimensional simulations of gas assisted moulding in complex geometries (Polynkin et al., 2002, 2005).

2. Modelling and simulation

Fig. 1 illustrates the process and defines some notation. We consider a straight cylindrical tube $0 \leqslant z \leqslant L$, $0 \leqslant r \leqslant R$,

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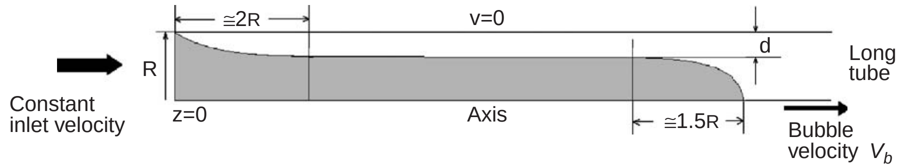

Fig. 1. Gas displacing a viscous shear thinning liquid from an axisymmetric tube. Indication of the development length and gas bubble leading interface form for an isothermal power law liquid with index $n = 0.7$. Definition of symbols, including the developed wall layer thickness d.

initially filled with a liquid. The flow properties of the liquid are described by a temperature-dependent truncated power law model that avoids unbounded viscosities at zero shear rate. The lower limiting shear rate is not a critical parameter in the present work—see further comments below—and the power law is chosen because it often represents shear thinning behaviour adequately over relevant shear rate ranges in terms of a single quantity, the power law index n.

$$\begin{array}{ll} \mu = K\dot{\gamma}^{n-1} & \dot{\gamma} > \dot{\gamma}_{\min} \\ \dot{\gamma} < \dot{\gamma}_{\min} & \dot{\gamma} = \dot{\gamma}_{\min} \end{array} \quad (1)$$

$$K = K_{\rm ref}\ \exp[-b(T - T_{\rm ref})].$$

The density, ρ, specific heat, C_p, and thermal conductivity, λ, of the liquid are constant. The viscosity and density of the gas are also constant.

An initial period of cooling is modelled, while the liquid is stationary.

$$t = 0, \quad 0 \leqslant z \leqslant l, \quad 0 \leqslant r \leqslant R, \quad T = T_0, \quad (2a)$$

$$0 < t \leqslant t_C, \quad 0 \leqslant z \leqslant l, \quad r = R, \quad T = T_C, \quad (2b)$$

$$\text{or} - \lambda T_{,r} = h(T - T_C). \quad (2c)$$

Here, h is a heat transfer coefficient governing the rate of heat transfer to a coolant at temperature T_C. Cooling is modelled using the axisymmetric form of the energy equation. After the period of static cooling, t_C, a constant inflow of gas is introduced at the tube inlet, $z = 0$. Liquid is displaced down the tube, creating an open-ended column of gas and leaving a liquid layer on the wall. Gas penetration is rapid, so that no further significant cooling takes place; however, since the liquid viscosity is high, the Reynolds number is very low. Under these conditions, the wall layer thickness becomes constant after about one tube diameter, and it is this layer thickness that is reported here. The simulations were, however, continued until the gas had penetrated several diameters down the tube, to confirm that developed conditions were established.

The equations for momentum and mass conservation are applied over the whole domain, $0 \leqslant z \leqslant L$, $0 \leqslant r \leqslant R$. Boundary conditions for the flow specify the gas inlet velocity, the no-slip condition at the tube wall, and developed flow conditions at the exit.

The problem domain was discretized using meshes of linear quadrilateral finite elements and solved using the commercial code FIDAP. Twenty elements were used to span the tube radius, with the element to element radial dimension decreasing towards the wall in the ratio 0.85. Convergence studies using meshes with up to 40 elements spanning the radius indicated that errors in locating the dimensionless radial position of the gas–liquid interface were below 3%. Further details of the flow simulation are provided in Polynkin et al. (2004). Although a series solution exists for the static conduction problem during the gas delay, it is more convenient to obtain the results numerically. An implicit first-order backwards difference scheme was used, with a constant time step size determined by convergence studies and comparison with the analytic solution.

In the absence of gravity, surface tension and inertia effects, the dimensionless developed residual layer thickness, δ, is a function of n, the power law index, which measures the degree of shear thinning, and the conditions at the end of the period of static cooling. These are characterized by the Fourier number, τ_C, corresponding to the period of static cooling; the Pearson number, Pn, which measures the effect of the imposed temperature change on viscosity; and the Biot number, Bi, which measures the relative magnitudes of convection heat flux at the boundary and conduction within the fluid. Eq. (3) defines these quantities.

$$\tau_C = \frac{\lambda t_c}{\rho C_p R^2}, \quad Pn = b(T_0 - T_C), \quad (3)$$

$$Bi = \frac{hR}{\lambda}, \quad \delta = \frac{d}{R}.$$

A further dimensionless group that arises in the non-isothermal flow of the liquid as it is displaced by the gas, is the Nahme group, Na. The product of this with the dimensionless flow time for displacement by the gas, τ_F, measures the fractional change in viscosity brought about by an adiabatic temperature rise due to irreversible viscous energy dissipation. These quantities are defined in Eq. (4).

$$\tau_F = \frac{\lambda t_F}{\rho C_P R^2}, \quad Na = \frac{K_{\rm ref} b R^2}{\lambda}(v/R)^{1+n}. \quad (4)$$

Here v is the mean liquid flow velocity and t_F the flow time. In many practical cases the product $Na\,\tau_F$ is small and the effects of viscous heating can be neglected. The group is therefore not included in the correlations that follow. Numerical results have shown that solutions are insensitive to the lower limiting shear rate in the power law model, Eq. (1), within two orders of magnitude about the value

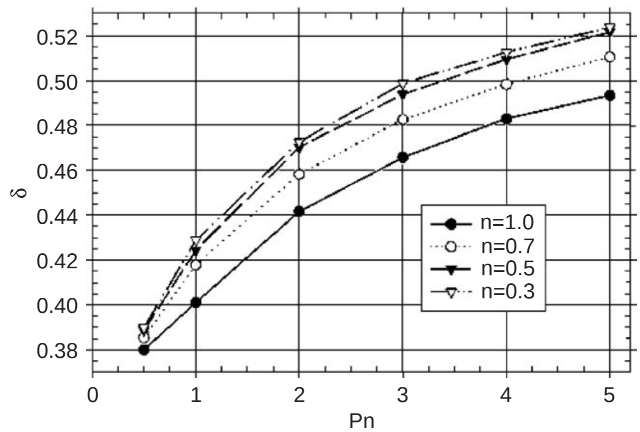

Fig. 2. Dependence of the dimensionless wall layer thickness δ on Pearson number *Pn* and power law index *n* after a dimensionless cooling time $\tau_C = 0.01$. Dirichlet temperature boundary conditions.

used, so $\dot{\gamma}_{\min}$ is also not included as a relevant parameter. This point is further discussed in Polynkin et al. (2004).

3. Results and discussion

3.1. Dirichlet temperature boundary conditions

Eighty production runs were carried out using the Dirichlet thermal boundary condition, Eq. (2b), each consisting of a cooling and a flow simulation, covering ranges of the dimensionless groups relevant to GAIM, $0.3 \leqslant n \leqslant 1.0$, $0.001 \leqslant \tau \leqslant 0.3$, $0.5 \leqslant Pn \leqslant 5.0$. Figs. 2 and 3 illustrate some of these results, showing how the deposited layer thickness increases with *Pn* and τ, the effect being more pronounced for low *n*. A consequence of this, as shown in Fig. 3, is that for short cooling times the layer thickness falls with decreasing *n*, as is well established for the isothermal case, see for example, Polynkin et al. (2004), but for longer cooling times it increases. The physics underlying this is as follows. The ability of the gas to displace liquid near the tube wall is related to the magnitude of the viscosity of the liquid near the wall relative to that nearer the axis of the tube. In isothermal flow of a shear thinning liquid, the higher shear rate near the wall reduces viscosity, resulting in a reduced layer thickness. When liquid near the wall is cooled, its viscosity increases, reducing shear rate next to the wall and producing a compensating increase closer to the axis. For shear thinning liquids, this increased shear rate reduces viscosity, magnifying the difference between viscosity at the wall and at a smaller radius, thus intensifying the effect of cooling in increasing layer thickness.

Layer thickness for the isothermal case was correlated with *n* in Polynkin et al. (2004) as follows:

$$\delta_{\text{Iso}} = \frac{0.404n}{0.086 + n}. \tag{5}$$

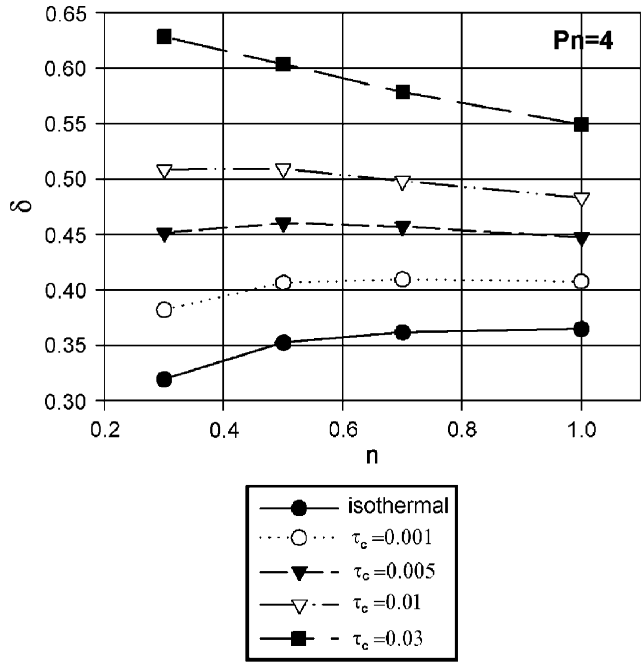

Fig. 3. Dependence of the dimensionless wall layer thickness δ on power law index *n* and dimensionless cooling time τ_C for Pearson number $Pn = 4.0$. Dirichlet temperature boundary conditions.

The effects of cooling may be expressed as an increment on this using the following formulae:

$$\delta_{\text{Dir}} = \delta_{\text{Iso}} + a(n)\tau_C{}^{b(n)} Pn^{c(n)}, \tag{6}$$

$$a(n) = 1.611 - 2.367n + 1.092n^2,$$

$$b(n) = 0.481 + 0.020n - 0.066n^2,$$

$$c(n) = 0.283 + 0.386n.$$

Eq. (6) reproduces the computed data points within 1–9%, the largest discrepancies occurring for high *Pn*. It is interesting to note that the dependence on time, $0.44 \leqslant b(n) \leqslant 0.48$, $n \geqslant 1 \geqslant 0$, is quite close to the square root dependence shown by thermal penetration depth according to the analytic solution for transient heat conduction.

3.2. Convection heat transfer boundary conditions

A more realistic boundary condition for the cooling is the use of a heat transfer coefficient, Eq. (2c). Two hundred and forty production runs were carried out for all combinations of $n = 0.3, 0.5, 0.7, 1.0$; $\tau_C = 0.01, 0.02, 0.03, 0.04, 0.05$; $Pn = 1.0, 2.0, 3.0, 4.0$; and $Bi = 1.0, 2.5, 20.0$, each consisting of a cooling and a flow simulation.

3.3. A Unified correlation $0 \leqslant Bi \to \infty$

A Biot number of zero corresponds to isothermal conditions, results for which are correlated by Eq. (5). Biot

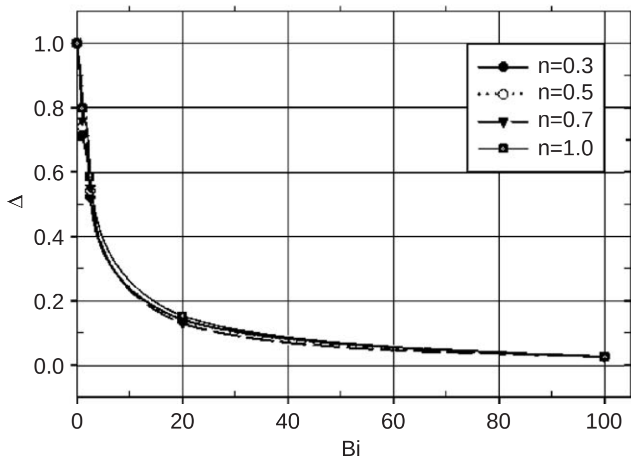

Fig. 4. Dependence of the reduced scaled wall layer thickness Δ as a function of Biot number Bi and power law index n, compiled from all results: isothermal, $Bi = 0.0$; intermediate Bi corresponding to convection boundary conditions; Dirichlet boundary conditions ($Bi \to \infty$) plotted at $Bi = 100$.

tending to infinity corresponds to the Dirichlet boundary conditions, as in Section 3.1. We find that within the parameter ranges investigated it is possible to interpolate within these limiting cases using the following correlation:

$$\delta_{Bi} = \delta_{\mathrm{Dir}}\left(1 - \frac{1}{1 + A(n)Bi^{B}}\right) + \delta_{\mathrm{Iso}}\left(\frac{1}{1 + A(n)Bi^{B}}\right) \quad (7)$$

$$A(n) = 0.458 - 0.209n, \quad B = 0.961$$

Eq. (7) represents all 240 data points with an average departure of 5% and a maximum of 10%, and the accuracy of the individual simulation results is believed to be well within these bounds. A further interesting quantity is obtained as

$$\Delta = \frac{\delta_{\mathrm{Dir}} - \delta_{Bi}}{\delta_{\mathrm{Dir}} - \delta_{\mathrm{Iso}}} = \frac{1}{1 + A(n)Bi^{B}}, \quad (8)$$

$$\Delta = 1, \quad Bi = 0, \quad \Delta = 0, \quad Bi \to \infty$$

Fig. 4 demonstrates the remarkably weak dependence of Δ on n.

4. Conclusion

Eqs. (5), (6) and (7) together provide predictions of the thickness of the wall layer left behind when gas displaces a viscous power law liquid from a cylindrical tube, for isothermal conditions or following a period of cooling. The correlations apply in the limit of high capillary number and low Reynolds number, and are of sufficient accuracy (within 10%) for engineering purposes. They are valid over ranges of the Pearson number, the Biot number and the Fourier number for cooling relevant in gas assisted injection moulding, for which technology they will useful in predicting outcomes under given processing conditions. They may also find application in other technologies, as well as provide insights into a complex non-isothermal two-phase flow problem.

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