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A theoretical bubble breakup model for slurry beds or three-phase fluidized beds under high pressure

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Abstract

A theoretical model for bubble breakup in slurry bubble columns as well as three-phase fluidized beds with fine particles has been developed based on an exploration into the deformation, oscillation and breakup process of the bubbles. The time response of the bubble to the bombarding eddy is taken into account, and as a result, the influence of the dispersed phase density and the operating pressure can be included in the model. The solids and the liquid phase are treated as a homogeneous mixture and the effect of solid concentration on turbulent properties is considered. The relationship between daughter size distribution and turbulent eddy size was established. The model predicts the tendency of decreasing bubble diameter under elevated pressure correctly.
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Keywords: Bubble breakup; Daughter size distribution; Bubble dynamics; High pressure; Slurry bed; Three-phase fluidized bed

1. Introduction

Slurry bubble columns as well as three-phase fluidized beds are widely applied in chemical and process industries due to their simplicity in construction, low operating cost, excellent heat and mass transfer characteristics and variable residence time. Typical applications of slurry bubble columns or three-phase fluidized-beds are found in processes such as hydrogenation and hydrodesulphurization of residual oil, waste-water treatment, fermentation, the Fischer–Tropch process, coal liquefaction, and hydrogenation of unsaturated fat and methanation of CO (Fan and Tsuchiya, 1989). However, despite their wide use in various industrial processes, many important aspects of the hydrodynamics in gas–liquid–solid three-phase flow related to these processes are still poorly understood.

The bubble size distribution, one of the most important parameters for reactor simulation and design, is related to the phase holdup, interaction between the phases and mass transfer behavior. Many works have employed the population balance model (PBM) to predict bubble diameters in such gas–liquid–solid three-phase flow or slurry bed systems. PBM was first formulated for chemical engineering purposes by Hulburt and Katz (1964), and has drawn unprecedented attention from both academic and industrial communities during the past few years because of its applicability to such bubbly flow systems and a wide variety of particulate processes (Ramkrishna and Mahoney, 2002). However, major problems remain in how to generalize the breakup and coalescence models and how to express them as functions of basic hydrodynamic parameters and physical properties.

In particular, bubble size distribution is mostly investigated at ambient conditions and for aqueous systems in spite of the fact that most industrial bubble columns are often operated at increased temperature and pressure while using organic liquids. Among the few studies considering significant pressure effects on bubble behavior (Lin et al., 1998; Fan et al., 1999), a correlation based on stress balance has been proposed to predict the maximum stable bubble size in the high pressure slurry bubble columns (Luo et al., 1999). However, the description of bubble behavior in such systems is far from being complete. A complete breakup kernel function should consist of both the breakup rate and the daughter size distribution. Furthermore, both the stress balance and the surface energy increase should

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be considered in the bubble breakup criteria. This work, therefore, aims to develop such a description for slurry beds or three-phase fluidized beds with low concentration of fine particles.

2. Model formulation

Previous studies on the breakup of bubbles in liquid phase (Prince and Blanch, 1990; Wilkinson et al., 1993; Luo and Svendsen, 1996) have revealed that the increase of surface energy during the breakup of bubbles is extracted from the continuous phase turbulent fluctuation. Our model is also based on this physical picture and, to make the problem tractable, the following assumptions are made:

(1) For both the liquid and the solid phases, interactions with the gas phase can be expressed as force acting on a bubble through the interface. In this case, the liquid–solid phase is treated as a pseudo-homogeneous medium especially when the particle size is much smaller compared to the bubble size, and the concentration of the solids is not too high. Then, the mixture parameters of those, such as the physical property and average viscosity, can be calculated based on well-established correlations (Mendes and Qassim, 1984; Tsuchiya et al., 1997):

$$\rho_c = (1-\alpha_p)\cdot\rho_l + \alpha_p\cdot\rho_p, \tag{1}$$

$$\mu_c = \mu_l \mathrm{e}^{(36.15\cdot\alpha_p^{2.5})}. \tag{2}$$

When particle size is in the order of the bubble size, the particle–bubble interaction plays an important role in the bubble breakup process. Then, the liquid–solid could not be treated as a pseudo-homogeneous phase (Chen and Fan, 1989).

(2) The continuous phase turbulence can be regarded as locally homogeneous and isotropic (Lee et al., 1987). Theoretical considerations and experimental evidence have shown (Hinze, 1959) that the fine-scale structure of most actual anisotropic turbulent flows is locally nearly isotropic. Thus, many features of isotropic turbulence may be applied to phenomena in actual turbulence that is determined mainly by the fine-scale structure. Furthermore, even actual turbulence with large-scale anisotropic structure or anisotropy on an essential part of its spectrum, can often be treated as isotropic turbulence approximated. The differences between the results are often sufficiently small to be disregarded compared to the uncertainty of the experimental data (Hinze, 1959).

(3) The breakup of a bubble in a turbulent flow field is due to the bombarding of eddies with characteristic size equal to or smaller than the bubble onto the surface of the bubbles. Eddies larger than the bubble merely carry the bubbles (Walter and Blanch, 1986; Lee et al., 1987; Luo and Svendsen, 1996).

(4) Only the binary breakup into two daughter bubbles is considered since it is overwhelmingly the major breakup manner supported by experimental observations (Walter and Blanch, 1986; Hesketh et al., 1991; Wilkinson et al., 1993).

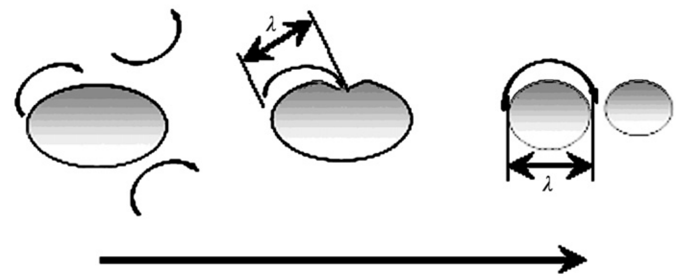

Fig. 1. Daughter size distribution under eddy bombarding.

2.1. Collision frequency

Following arguments from the kinetic theory of gases, Prince and Blanch (1990) postulated in their model that the collision frequency for eddies within a unit size interval around λ with bubbles of size d_b can be expressed as

$$\omega(d_b,\lambda) = \frac{\pi}{4}(d_b+\lambda)^2 n_\lambda \overline{u}_\lambda n_b. \tag{3}$$

According to the isotropic turbulent assumption, the number density (n_λ) of the eddies around the sizes of λ for unit size interval can be expressed as (Tennekes and Lumley, 1972)

$$n_\lambda = \frac{0.822(1-\alpha_d)}{\lambda^4}, \tag{4}$$

and the mean turbulent velocity ($\overline{u}_\lambda$) of the eddies with size λ can be expressed as (Kolmogorov, 1949; Hinze, 1955)

$$\overline{u}_\lambda = \sqrt{2}(\varepsilon\lambda)^{1/3}. \tag{5}$$

2.2. Daughter size distribution

The sizes of the bubbles formed in the breakup process, the so-called daughter size distribution, must be included in a complete bubble breakup model. Historically, there have been three predominant approaches to the formulation of this term: statistical models (Novikov and Dommermuth, 1997), phenomenological models based on the change in surface energy of a breaking bubble (Prince and Blanch, 1990; Martinez-Bazan et al., 1999), and hybrid models which are based on a combination of both (Konno et al., 1980). A phenomenological model with consideration to surface energy increasing and capillary pressure constraints was formulated in this work.

Since the distortion of the bubble is caused by velocity fluctuations over a certain distance (the so-called eddy), it is reasonable to assume that one of the daughter bubble formed in the breakup has the characteristic size of the original eddy, as shown in Fig. 1.

Then the daughter bubble size distribution can be characterized by the volumetric ratio of a daughter bubble to its "mother",

f_{BV}. In this case, one of the daughters will have

$$f_{BV} = \left(\frac{\lambda}{d_b}\right)^3, \tag{6}$$

with the other having $1 - f_{BV}$.

2.3. Eddy efficiency

The bubbles are usually deformed by the turbulent flow field and stretched in one direction when eddies bombard on bubble surface. This leads to a necking that contracts further, resulting finally in the breakage. The surface energy increased during bubble deformation is converted from the kinetic energy contained in the eddies. The bubble also oscillates under the bombarding of eddies in its own frequency. However, most bubble breakup models neglect the dynamics of the bubbles. Levich (1962) and Hesketh et al. (1991) reported that the fluctuation frequency of the bubble surface, $f(n)$, in developed turbulent flow field can be expressed as

$$f(n) = \left[\left(\frac{2\sigma}{\pi^2 d_b^3}\right)\left(\frac{(n+2)(n+1)n(n-1)}{(n+1)\rho_d + n\rho_c}\right)\right]^{1/2}, \tag{7}$$

where the $n = 2$ mode is for a shape oscillation starting from a spherical shape and passing through oblate, spherical, and prolate shapes, which is in close resemblance to actual bubble oscillations. The temporal response of the bubble controls the maximum amount of energy which can be extracted from each turbulent eddy and it can be determined from the contact time between the bubble and the eddy.

According to Levich (1962), the characteristic time of the eddies in turbulent flow is

$$\tau_e = \frac{\lambda^{2/3}}{\varepsilon^{1/3}}. \tag{8}$$

When the eddy duration is less than half of the bubble oscillation period, the bubble deforms continuously and extracts the kinetic energy exhaustively from the dissipating eddy. On the other hand, for longer eddy durations, the bubble is bounced away with some kinetic energy remains in the eddy. Though the hope to fully quantify the time evolution of this bombarding process is still remote, it may be reasonable, as a rough guess, to assume that bubble surface area increases linearly (Risso and Fabre, 1998) in most part of the process. In fact, we can expect that, after bombarding on the bubble surface, the eddy suffers increasing resistance from the bubble while the velocity of the eddy decreases with increasing contact time; these contrary tendencies may result in a relatively constant energy transfer rate (of course, it is not true near the bouncing point where the variation of surface area must be zero). Then, the ratio between the half period of bubble oscillation and the eddy duration can be defined as the eddy efficiency:

$$C_{\text{eddy}} = \min(0.5/(f(2)\tau_e), 1). \tag{9}$$

It can also be considered as the portion of the energy that can be extracted from the eddy and converted to the surface energy of the bubble during its deformation.

2.4. Breakup probability and breakup rate

The eddies in turbulent flow fields arrive at the surface of bubbles with different levels of kinetic energy. The exponential energy density function is suitable to describe this distribution according to some previous researches (Kuboi et al., 1972):

$$P_e(\chi) = \frac{1}{\bar{e}(\lambda)}\exp(-\chi), \quad \chi = \frac{e(\lambda)}{\bar{e}(\lambda)}, \tag{10}$$

where the kinetic energy of an eddy with size λ, $e(\lambda)$, is defined as

$$e(\lambda) = \rho_c \frac{\pi}{6}\lambda^3 \frac{u_\lambda^2}{2}, \tag{11}$$

and $\bar{e}(\lambda)$ is the mean kinetic energy of eddies with characteristic size λ, which can be calculated from the mean turbulent velocity ($\bar{u}_\lambda$) of the eddies given in Eq. (5).

The analysis in Section 2.3 suggests that a bubble breaks only when the kinetic energy of the bombarding eddy exceeds the increase in surface energy required for the breakage, which is in agreement with some previous researches (Luo and Svendsen, 1996) showing that

$$e(\lambda) \geqslant \frac{c_{f_{BV}} \pi d_b^2 \sigma}{C_{\text{eddy}}}. \tag{12}$$

The difference in surface energy can be calculated from the coefficient of surface area increase in the breakage of a single bubble, $c_{f_{BV}}$, which can be expressed as

$$c_{f_{BV}} = \left(\frac{\lambda}{d_b}\right)^2 + \left[1 - \left(\frac{\lambda}{d_b}\right)^3\right]^{2/3} - 1. \tag{13}$$

Moreover, when the breakage produces a very small bubble, that is, with a very small f_{BV}, its high capillary pressure will present additional resistance to its deformation, which may explain the singularity occurs at small $c_{f_{BV}}$. To take full consideration of this effect, Lehr and Millies (2002) proposed that

$$\frac{1}{2}\rho_c u_\lambda^2 > 2\frac{\sigma}{d_b \cdot \min(f_{BV}, 1 - f_{BV})^{1/3}} \tag{14}$$

and hence, the minimum energy an eddy should contain to break a bubble is

$$e_c(d_b, \lambda) = \max\left(\frac{c_{f_{BV}} \pi d_b^2 \sigma}{C_{\text{eddy}}}, \frac{\pi\sigma\lambda^3}{3d_b \cdot \min(f_{BV}, 1 - f_{BV})^{1/3}}\right). \tag{15}$$

Consequently, the breakage probability of the bubble when hit by the eddy should be equal to the probability of the eddy having a kinetic energy no less than the minimum energy required for the bubble to break up. That is,

$$P_B(d_b, \lambda) = P_e[e(\lambda) \geqslant e_c(d_b, \lambda)]. \tag{16}$$

Under the bombarding of eddies in a unit size interval, the corresponding breakup frequency of the bubble can be expressed as the bubble–eddy collision rate (ω) multiplied by this breakup probability, that is,

$$b(d_b, \lambda) = \omega(d_b, \lambda) \cdot P_B(d_b, \lambda). \quad (17)$$

The total breakup frequency of the bubble can be obtained by integrating this frequency from the minimum eddy size in the entire inertial sub-range ($\lambda_{\min}$) to d_b:

$$\Omega(d_b) = \int_{\lambda_{\min}}^{d_b} \omega(d_b, \lambda) \cdot P_B(d_b, \lambda)\, d\lambda, \quad (18)$$

where $\lambda_{\min}$ takes the value of 11.4 times the Kolmogorov length scale (Tennekes and Lumley, 1972):

$$\lambda_{\min} = 11.4 \cdot \frac{(\mu_c/\rho_c)^{0.75}}{\varepsilon^{0.25}}. \quad (19)$$

From here we can conclude that the particle phase could influence the continuous phase properties and then the turbulent interactions between the continuous phase and the bubbles may change.

Daughter size distribution can be gained from the model described above. The daughter particle size distribution, $\beta(d_b, f_{BV})$, was first introduced by Valentas et al. (1966) to describe the size distribution of daughter drops or bubbles. For a continuous daughter size distribution, $\beta(d_b, f_{BV}) \cdot \Delta f_{BV}$ represents the fraction of bubbles of diameter d_b that break into bubbles of volumetric ratio between f_{BV} and $f_{BV} + \Delta f_{BV}$. The dimensionless daughter size distribution, $\beta(d_b, f_{BV})$, is then expressed as

$$\beta(d_b, f_{BV}) = \frac{d_b \cdot b(d_b, \lambda)}{3 f_{BV}^{2/3} \cdot \Omega(d_b)} + \frac{d_b \cdot b(d_b, d_{b-\lambda})}{3(1 - f_{BV})^{2/3} \cdot \Omega(d_b)} \quad (20)$$

with $\lambda = d_b \cdot f_{BV}^{1/3}$.

3. Results and discussion

So far, a bubble breakup kernel function has been formulated theoretically, which is free of unknowns or tuning parameters. In this model, the bubble dynamics was taken into account to evaluate the influence of dispersed phase density or operating pressure. Both the breakage rate and the daughter bubble size distribution can be predicted with given operating conditions and fluid properties. The results for typical systems under different conditions are analyzed in the following.

3.1. The influence of pressure

As mentioned above, bubble breakup is controlled by the extractable eddy kinetic energy and the capillary pressure of the smaller daughter bubble. Their effects are shown in Fig. 2.

Region I in Fig. 2 shows that for small eddies whose turnover time is less than half of the bubble oscillation period, kinetic energy contained in the eddies can be fully extracted by the bubble. But due to the low kinetic energy contained in small eddies, the bubble breakup frequency is nearly zero. With the increase of eddy size, its turnover time increases simultaneously,

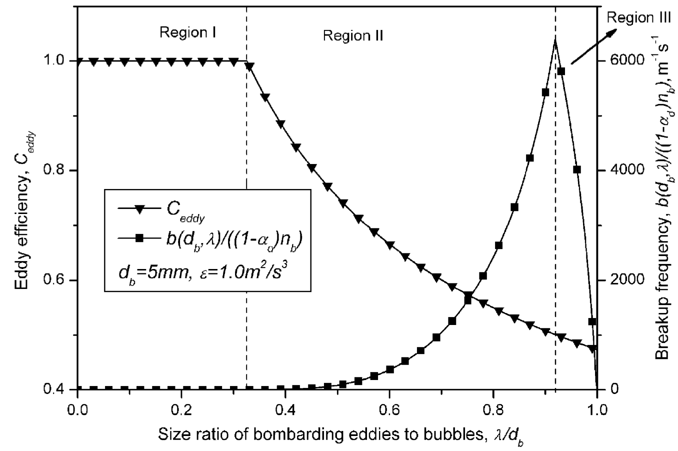

Fig. 2. Eddy efficiency and breakup probability vs. bombarding eddy size ($\rho_l = 998\,\mathrm{kg\,m^{-3}}$, $\rho_d = 1.2\,\mathrm{kg\,m^{-3}}$, $\rho_p = 2500\,\mathrm{kg\,m^{-3}}$, $\sigma = 0.072\,\mathrm{N\,m^{-1}}$, $\mu_l = 0.001\,\mathrm{Pa\,s}$, $d_p = 0.5\,\mathrm{mm}$, $\alpha_d = 0.05$, $\alpha_p = 0.05$).

and the maximum percent of energy that can be converted to the bubble surface energy decreases. However, because of the increase of kinetic energy contained in larger eddies, the bubble breakup frequency becomes higher. Region II in Fig. 2 shows this kind of trend. In region III, the breakup of bubbles under the bombarding of eddies with size close to the bubble diameter could produce very small bubbles. Due to the high capillary pressure inside small bubbles, the arriving eddy needs to have high dynamic pressure in order to break the bubbles. So, the breakup frequency decreases with the increasing of arriving eddy size in region III.

There are many industrial processes operating under elevated pressures. Many experimental investigations show that the operating pressure (or the gas density) has a significant influence on the bubble diameter in slurry bubble columns or three-phase fluidized beds (Fan and Tsuchiya, 1989). It is, therefore, reasonable to expect that the changes in dispersed gas phase density induced by elevated pressure could have an influence on the bubble breakup rate. With the increase of dispersed gas phase density, the specific breakup rate increases simultaneously. This should be a result of the increase in bubble oscillation period. More kinetic energy could be extracted from arriving eddies when the bubble oscillation period becomes longer. Bubbles with higher gas density breakup much easier, and smaller bubbles could be obtained consequently. Fig. 3 shows the influence of the dispersed phase density on the breakup rate of bubbles with a specific size.

3.2. Breakup rate

Few experiments have been conducted to determine the breakup rate of bubbles in three-phase fluidized beds. For gas–liquid two-phase flow, Lasheras et al. (1999) studied the breakup of air bubbles in a turbulent water jet. The LDA technique is used to measure the turbulent energy spectrum in the jet to calculate the local dissipation rate. By calculating the largest bubble size decrease due to breakup, Lehr and Millies

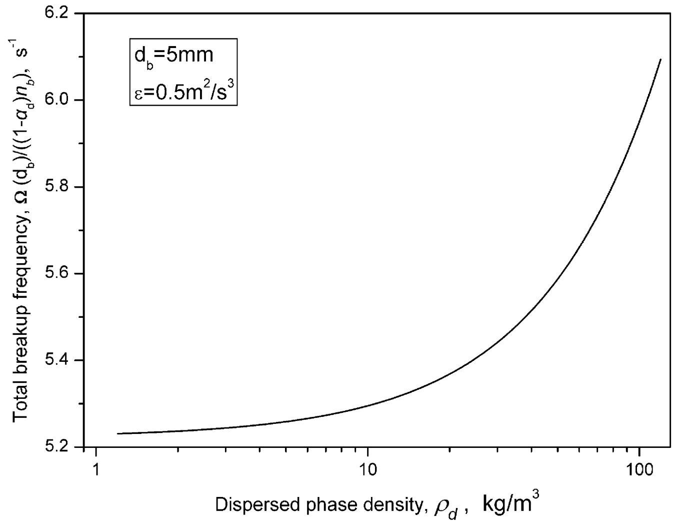

Fig. 3. Breakup frequency vs. various dispersed phase density (ρ_l = 998 kg m^{-3}, ρ_p=2500 kg m^{-3}, σ=0.072 N m^{-1}, μ_l=0.001 Pa s, d_p=0.5 mm, α_d = 0.05, α_p = 0.05).

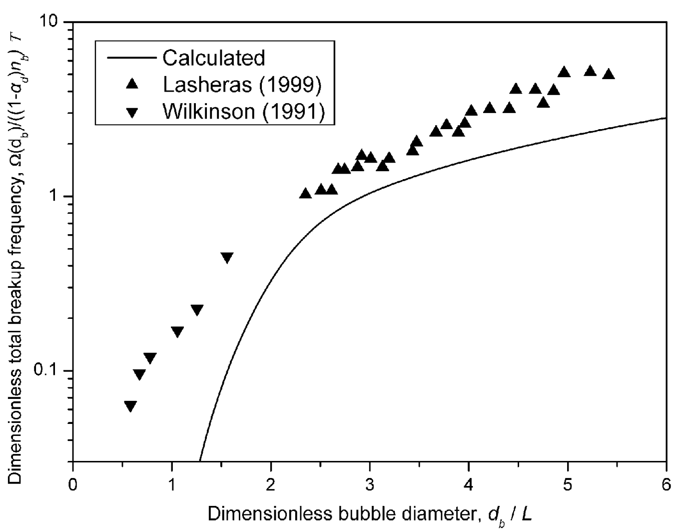

Fig. 4. Predicted breakup frequency of the bubbles vs. experimental results in air–water systems.

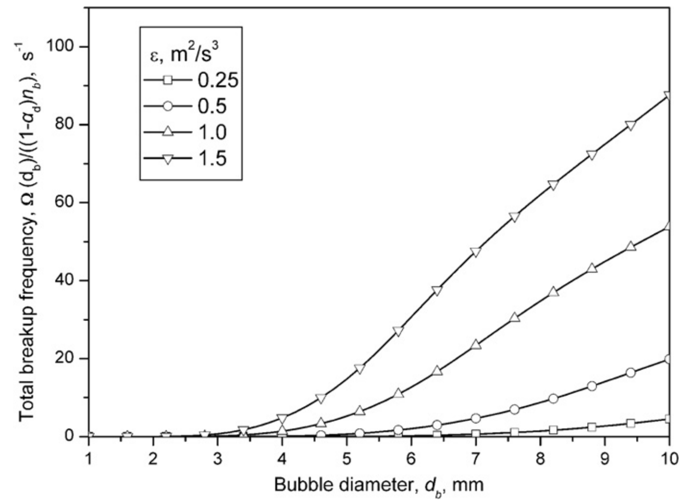

Fig. 5. Influence of energy dissipation rate on the breakup rate of different size bubbles (ρ_l = 998 kg m^{-3}, ρ_d = 1.2 kg m^{-3}, ρ_p = 2500 kg m^{-3}, σ = 0.072 N m^{-1}, μ_l = 0.001 Pa s, α_d = 0.05, d_p = 0.5 mm, α_p = 0.05).

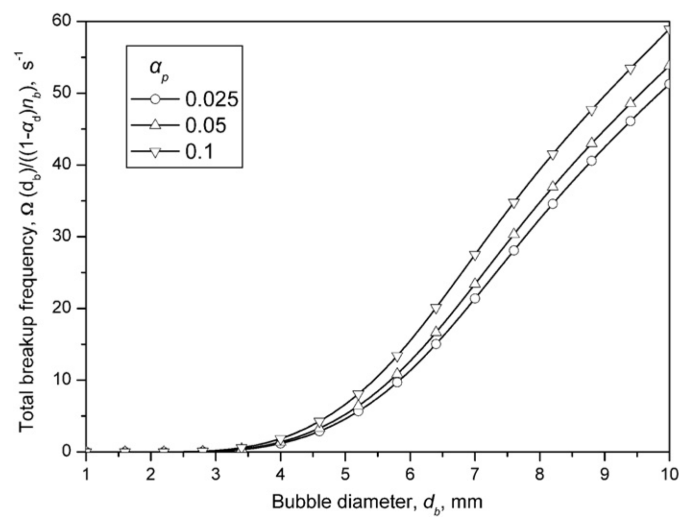

Fig. 6. Influence of solid phase volume fraction on the breakup rate of different size bubbles (ρ_l = 998 kg m^{-3}, ρ_d = 1.2 kg m^{-3}, ρ_p = 2500 kg m^{-3}, σ = 0.072 N m^{-1}, μ_l = 0.001 Pa s, α_d = 0.05, d_p = 0.5 mm, ε = 0.5 m^2 s^{-3}).

(2002) established the connection between bubbles breakup frequency and the dissipation rate. These results are shown in Fig. 4, as well as that from Wilkinson (1991) for bubbles in turbulent pipe flow and from the predictions of our model. For the convenience of comparison, all the results are made dimensionless using the following length and time scales:

$$L = \left(\frac{\sigma}{\rho_c}\right)^{3/5} \frac{1}{\varepsilon^{2/5}}, \tag{21}$$

$$T = \left(\frac{\sigma}{\rho_c}\right)^{2/5} \frac{1}{\varepsilon^{3/5}}. \tag{22}$$

It can be found that our model shows good agreement with available experimental measurement.

Bubbles of different sizes behave differently under the bombarding of turbulent eddies. With the increase of mother bubble size, the number of eddies with size equal to or smaller than this bubble increases and leads to an increase of the bubble collision frequency and the breakup rate. Furthermore, with the increase of mother bubble size, the efficiency of the eddy of certain size increases due to the increase of bubble oscillation period. Increase in the energy dissipation rate also leads to considerably more frequent collisions between bubbles and eddies. Moreover, the kinetic energy contained in an eddy also increases with the increase of energy dissipation rate. The effects can be seen in Fig. 5 clearly.

With the presence of small particles in the liquid phase, the breakup frequency of the bubble becomes a little bit higher, as shown in Fig. 6. But we cannot conclude that the mean

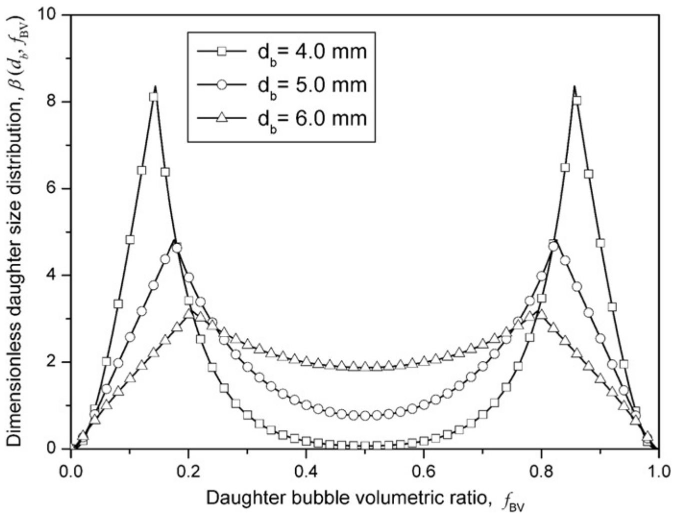

Fig. 7. Daughter size distributions of different size bubbles ($\rho_l = 998\,\mathrm{kg\,m^{-3}}$, $\rho_d = 1.2\,\mathrm{kg\,m^{-3}}$, $\rho_p = 2500\,\mathrm{kg\,m^{-3}}$, $\sigma = 0.072\,\mathrm{N\,m^{-1}}$, $\mu_l = 0.001\,\mathrm{Pa\,s}$, $\alpha_d = 0.05$, $d_p = 0.5\,\mathrm{mm}$, $\alpha_p = 0.05$, $\varepsilon = 0.5\,\mathrm{m^2\,s^{-3}}$).

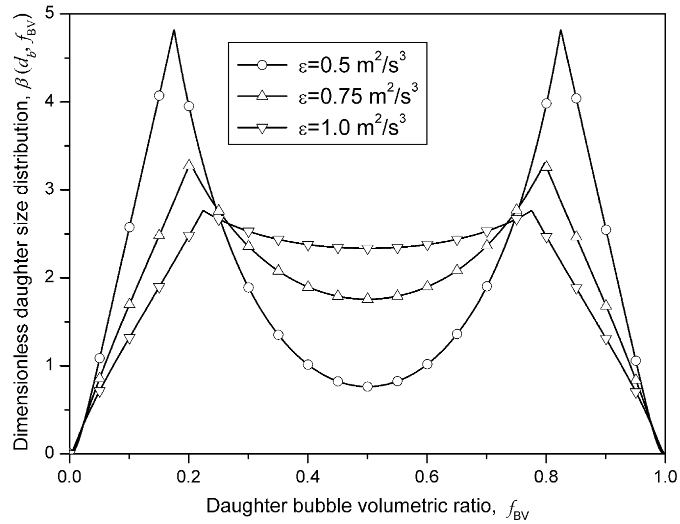

Fig. 8. Daughter size distribution of a bubble under different turbulent intensity ($\rho_l = 998\,\mathrm{kg\,m^{-3}}$, $\rho_d = 1.2\,\mathrm{kg\,m^{-3}}$, $\rho_p = 2500\,\mathrm{kg\,m^{-3}}$, $\sigma = 0.072\,\mathrm{N\,m^{-1}}$, $\mu_l = 0.001\,\mathrm{Pa\,s}$, $\alpha_d = 0.05$, $d_p = 0.5\,\mathrm{mm}$, $\alpha_p = 0.05$, $d_b = 5\,\mathrm{mm}$).

bubble diameter will become smaller because the viscosity of the liquid–solid mixture will increase with the increase of the particle concentration, and then bubble coalescence will be promoted by this effect. The change of the mean bubble diameter will be determined by the competition between these two factors.

3.3. Daughter size distribution

Many experimental results give a U-shaped daughter size distribution, that is, unequal-sized breakup predominates bisections (e.g., Hesketh et al., 1991). This tendency is well predicted by our model as can be seen in Fig. 7. However, with the increase of the mother bubble diameter, more energy is needed for a certain f_{BV} breakup, but the energy contains in the eddies increases much faster than the increase of surface area difference between the mother bubble and the two smaller daughter bubbles. As a result, more flattened daughter size distribution can be obtained, which is also shown in Fig. 7.

As the effect of capillary pressure on the formation of small daughter bubble is considered in our model, a decreasing probability is predicted when one of the daughter bubble volume falls below a certain small value, which can be seen in Figs. 7 and 8. This is in accordance with the underlying physical picture. While in many other models (e.g., Tsouris and Tavlarides, 1994; Luo and Svendsen, 1996), the probability goes to a maximum when the breakup fraction tends to zero, which is contrary to the fact that the capillary pressure on the bubble surface is extremely high when its radius of curvature approaches zero, and hence hardly possible for the formation of tiny daughter bubbles.

4. Conclusion

A theoretical bubble breakup rate model for slurry beds or three-phase fluidized beds has been developed based on existing theories of probability and turbulence, with considerations for both energy constraints and force balance constraints. The time response of the bubble to the bombarding eddies is also taken into account, and as a result, the influence of the dispersed phase density and the operating pressure can be included in the model. The solids and the liquid phase are treated as a homogeneous mixture and the effect of solids concentration on turbulent properties is considered. The breakup model has no tuning parameters since all constants in the model are determined from isotropic turbulence theory.

The breakup frequency predicted by the present model for air–water systems are shown to be in good agreement with the experimental result of Lasheras et al. (1999) and Wilkinson (1991). Different aspects of the bubble breakup process are analyzed, which are found to be reasonable and consistent with existing theories or experimental observations. In particular, the intensification of bubble breakup under elevated pressure and higher solids concentration (for dilute suspension of fine particles) is predicted.

Notation

b	breakup frequency of a bubble at certain f_{BV}, $\mathrm{s^{-1}}$
$c_{f_{BV}}$	coefficient of surface area increment during bubble breakup
C_{eddy}	eddy efficiency
d_b	bubble diameter, m
e	kinetic energy of an eddy, $\mathrm{kg\,m^2\,s^{-2}}$
e_c	critical kinetic energy in an eddy to breakup a bubble, $\mathrm{kg\,m^2\,s^{-2}}$
f	bubble oscillation frequency, $\mathrm{s^{-1}}$
f_{BV}	volumetric ratio of a daughter bubble to the mother bubble
L	characteristic length, m
n	oscillation mode number

n_b number density of the bubbles, m^{-3}
n_λ number density of the eddies around the sizes of λ for unit size interval, m^{-4}
P_B breakup probability, m^{-4}
T characteristic time, s
$\bar{u}_\lambda$ mean turbulent velocity, $m\,s^{-1}$

Greek letters

α_d volume fraction of the gas phase
α_p volume fraction of the particle phase in the liquid–solid phase
β daughter bubble size distribution, m^{-3}
ε rate of energy dissipation in the liquid phase per unit mass, $m^2\,s^{-3}$
λ character size of an eddy, m
μ_c viscosity of the liquid–solid phase mixture, Pa s
μ_l liquid phase viscosity, Pa s
ρ_c density of the liquid–solid phase mixture, $kg\,m^{-3}$
ρ_d gas phase density, $kg\,m^{-3}$
ρ_p solid phase density, $kg\,m^{-3}$
σ surface tension, $N\,m^{-1}$
τ_e characteristic time of the eddies, s
Ω total breakup frequency of the bubbles, s^{-1}
ω collision frequency, s^{-1}

Acknowledgments

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