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A numerical study of momentum and forced convection heat transfer around two tandem circular cylinders at low Reynolds numbers. Part II: Forced convection heat transfer

Gheorghe Juncu *

Politehnica University Bucharest, Catedra Inginerie Chimica, Polizu 1, 78126 Bucharest, Romania

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Abstract

This work presents a computational study of the forced convection heat transfer around two circular cylinders in tandem. Axisymmetric, steady, laminar flow around the cylinders was assumed. The temperature inside the cylinders is considered spatially uniform but not constant in time. Numerical solutions have been obtained in bipolar cylindrical coordinates. The finite difference method was used to discretize the equations of the mathematical model. The influence of the model parameters on the heat transfer rate was analysed for the upstream cylinder Reynolds number, *Re*, varying from 1 to 30 and fluid phase Prandtl number equal to 0.1, 1, 10 and 100.
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Keywords: Heat transfer; Forced convection; Two circular cylinders; Tandem arrangement; Steady laminar flow; Bipolar cylindrical coordinates

1. Introduction

Compared with the isothermal flow past two cylinders in tandem [1], there is only one publication [2], to our knowledge, on the numerical solution of the heat transfer from two circular cylinders in tandem. Buyruk [2] studied the forced convection heat transfer for tandem, in-line and staggered cylinders configurations. The computations were carried out for constant fluid properties, incompressible fluid, laminar flow and steady state conditions. The ANSYS/Flotran CFD software was used. For equal size cylinders in tandem, the influence of the center-to-center distance on the heat transfer rate was analysed at $Re = 400$ and $Pr = 0.71$.

Related problems that may be mentioned in this section are (a) heat transfer around two spheres in tandem; (b) heat transfer from an infinite or finite array of in-line cylinders and (c) heat transfer from array of parallelepiped obstacles.

The heat transfer around two spheres in tandem at moderate *Re* numbers was investigated numerically in [3,4]. We will not insist here on this subject. It is discussed and analysed in detail in [5].

The steady forced convection heat transfer in a laminar flow field over an infinite (periodic) and finite in-line array of cylinders was studied numerically in [6,7] (and the references mentioned herein). In these numerical studies it is frequently assumed that the flow is steady and symmetric with respect to the centerline of the cylinders in the same row. Owing to the symmetries of the system, it was assumed that no momentum or energy transfer takes place in the cross-flow direction at the streamwise equidistant planes between rows of cylinders. Two truncated domains were studied in [6]: a single cylinder cell and a five-cylinder cell. The first was considered relevant for the developed regime while the second for the developing regime.

Young and Vafai [8] analysed numerically the convective flow and heat transfer in a channel containing multiple heated obstacles attached to one wall. The effects of variations in the obstacle height, width, spacing and number along with obstacle thermal conductivity, fluid flow rate

* Tel./fax: +40 21 345 05 96.
E-mail addresses: juncugh@netscape.net, juncu@easynet.ro

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Nomenclature

C_p heat capacity
d cylinder diameter
k thermal conductivity
L distance from the center of the cylinder to the origin of the coordinate system
Nu instantaneous Nusselt number
Pr fluid phase Prandtl number, $Pr = \rho_f C_{p,f} v_f / k_f$
Re Reynolds number based on diameter of the upstream cylinder, $Re = U_\infty d_1 / v$
t time
T temperature
U_∞ free stream fluid velocity
V volume
Z dimensionless temperature defined by the relation, $Z_{(c)} = \frac{T_{f,(c)} - T_{f,\infty}}{T_{c,0} - T_{f,\infty}}$

Greek symbols
η bipolar cylindrical coordinate
λ Nusselt numbers ratio (tandem cylinder)/(isolated cylinder)
v kinematic viscosity
ρ density
ξ bipolar cylindrical coordinate
Ξ volume heat capacity ratio $(\rho_c C_{p,c})/(\rho_f C_{p,f})$
τ dimensionless time or Fourier number, $\tau = 4tk_f/(\rho_f C_{p,f} d_1^2)$
ψ stream function

Subscripts
c refers to the cylinders
f refers to the fluid phase
0 initial conditions
1 refers to the upstream cylinder
2 refers to the downstream cylinder

and heating method were studied. The periodicity behaviour of the velocity components and temperature distributions were explicitly demonstrated. Conjugate heat transfer for three-dimensional developing turbulent flows over an array of cubes in cross-stream direction was studied numerically in [9]. Yaghoubi and Velayati [9] presented the heat transfer characteristics resulting from the recirculation zone around the blocks for a wide range of Reynolds numbers, $4.2\text{E}03 \leqslant Re \leqslant 1.0\text{E}05$, $Pr = 0.7$ and blockage ratios from 10% to 50%.

The present research is dedicated to the analysis of forced convection heat transfer from two tandem cylinders in a steady, laminar flow. The temperature inside the cylinders is considered spatially uniform but not constant in time. The Reynolds number based on the diameter of the leading cylinder, Re, takes value in the range $1 \leqslant Re \leqslant 30$. For each Re number, the values considered for the fluid phase Prandtl number, Pr, are $Pr = 0.1$, 1, 10 and 100. The main aspect investigated is the influence of Re, Pr and volume heat capacity ratio on the heat transfer rate for cylinders with the same diameter and identical physical properties.

2. Statement of the problem

The physical model of the present problem is discussed in detail in the first part of this work. To model the heat transfer between two cylinders in tandem the following supplementary assumptions were considered (the assumptions made in [1] also remain valid):

(i) during the heat transfer, the volume and the shape of the cylinders remain constant;
(ii) the cylinders have the same initial temperature;
(iii) the cylinders have the same physical properties;
(iv) the physical properties are constant;
(v) no phase change occurs during the heat transfer;
(vi) no chemical reaction inside the cylinders or in the surrounding fluid;
(vii) the effects of free convection, viscous dissipation and radiation are negligible;
(viii) at the interface, thermodynamic equilibrium is established instantaneously.

Under the previous assumptions, the heat balance equations for an axisymmetrical flow field in a general orthogonal curvilinear coordinates α, β, ϕ (axisymmetric versus the coordinate ϕ) are

- fluid phase

$$\rho_f C_{p,f}\left(\frac{\partial T_f}{\partial t} + v_\alpha \frac{1}{h_\alpha}\frac{\partial T_f}{\partial \alpha} + v_\beta \frac{1}{h_\beta}\frac{\partial T_f}{\partial \beta}\right) = k_f \frac{1}{h_\alpha h_\beta h_\phi}\left[\frac{\partial}{\partial \alpha}\left(\frac{h_\beta h_\phi}{h_\alpha}\frac{\partial T_f}{\partial \alpha}\right) + \frac{\partial}{\partial \beta}\left(\frac{h_\alpha h_\phi}{h_\beta}\frac{\partial T_f}{\partial \beta}\right)\right] \quad (1)$$

- cylinder

$$\rho_{c,i} C_{p,c,i} V_{c,i} \frac{\partial T_{c,i}}{\partial t} = k_f \int\int_{\sigma_i} \frac{1}{h_\alpha}\frac{\partial T_f}{\partial \alpha} h_\beta h_\phi \, d\beta \, d\phi, \quad i = 1,2 \quad (2)$$

where

$$v_\alpha = -\frac{1}{h_\beta h_\phi}\frac{\partial \psi}{\partial \beta}, \quad v_\beta = \frac{1}{h_\alpha h_\phi}\frac{\partial \psi}{\partial \alpha},$$

ψ is the stream function and σ_i the cylinder surfaces.

As in [1], for a pair of cylinders in tandem, the orthogonal bipolar cylindrical coordinate system [10] was used. The relation between the Cartesian coordinates (x, y, z) and the bipolar cylindrical coordinates (η, ξ, z) is,

$$x = \frac{c \sin \xi}{\cosh \eta - \cos \xi}, \quad y = \frac{c \sinh \eta}{\cosh \eta - \cos \xi}, \quad z = z,$$

where $c > 0$ is a characteristic length. The surfaces of the cylinders are located at $\eta = \eta_1$ $(\eta_1 < 0)$ and $\eta = \eta_2$ $(\eta_2 > 0)$. The relations between η_i, the diameters of the cylinders d_i and the distances L_i of their centers from the origin of the coordinates system are

$$\frac{d_i}{2} = \frac{c}{\sinh |\eta_i|}, \quad L_i = c \coth |\eta_i|, \quad i = 1, 2.$$

Note that if $d_1 = d_2 = d$, it results from the previous relations that $-\eta_1 = \eta_2$ and $L_1 = L_2 = L$. More discussions about the bipolar cylindrical coordinate can be viewed in [10]. The scale factors (metric coefficients) h_α, h_β, h_ϕ, for the bipolar cylindrical coordinate system are

$$h_\alpha = h_\eta = \frac{c}{\cosh \eta - \cos \xi}, \quad h_\beta = h_\xi = \frac{c}{\cosh \eta - \cos \xi},$$
$$h_\phi = 1.$$

We define the following dimensionless variables and groups (the radius of the upstream cylinder is considered as the length scale and the free stream velocity U_∞ as the velocity scale):

$$\bar{c} = \frac{2c}{d_1}, \quad Z_{(c)} = \frac{T_{(c)} - T_\infty}{T_{c,0} - T_\infty}, \quad \tau = \frac{4tk_f}{\rho_f C_{p,f} d_1^2}, \quad \bar{\psi} = \frac{4\psi}{U_\infty d_1^2}$$
$$Pr = \frac{\rho_f C_{p,f} \nu_f}{k_f}, \quad Re = \frac{U_\infty d_1}{\nu_f}, \quad \Xi = \frac{\rho_c C_{p,c}}{\rho_f C_{p,f}}.$$

After η and ξ are substituted for α and β in (1) and (2), the non-dimensional governing equations for the thermal energy are

- fluid phase

$$\frac{\partial Z}{\partial \tau} + \frac{Pr\,Re}{2} A^2 \left(\frac{\partial \bar{\psi}}{\partial \eta} \frac{\partial Z}{\partial \xi} - \frac{\partial \bar{\psi}}{\partial \xi} \frac{\partial Z}{\partial \eta} \right) = A^2 \left(\frac{\partial^2 Z}{\partial \eta^2} + \frac{\partial^2 Z}{\partial \xi^2} \right) \quad (3a)$$

- cylinder

$$\left(\frac{d_i}{d_1} \right)^2 \frac{dZ_{c,i}}{d\tau} = -\frac{2}{\pi} \Xi_i^{-1} \int_0^\pi \left. \frac{\partial Z}{\partial \eta} \right|_{\eta = \eta_i} d\xi, \quad i = 1, 2, \quad (3b)$$

Table 1
Steady state values of the Nusselt number for cylinders with constant temperature at $d_1 = d_2 = d$ and $2L/d = 2$

Re	Pr	Nu_1	Nu_2	Nu (isolated cylinder)
1	0.1	0.3384	0.2812	0.5031
	1	0.6988	0.4525	0.8774
	10	1.4749	0.8137	1.6672
	100	3.0386	1.6224	3.3557
2	0.1	0.4252	0.3218	0.5988
	1	0.9434	0.5490	1.110
	10	2.0011	1.0304	2.1841
	100	4.1320	2.0884	4.4736
5	0.1	0.6173	0.4011	0.7825
	1	1.4210	0.7311	1.5611
	10	3.0136	1.4307	3.1797
	100	6.2572	2.9194	6.6051
10	0.1	0.8384	0.4846	0.9850
	1	1.9383	0.9279	2.0560
	10	4.1065	1.8459	4.2572
	100	8.5924	3.7464	8.8741
15	0.1	1.0061	0.5477	1.1382
	1	2.3199	1.0738	2.4252
	10	4.9152	2.1456	5.0562
	100	10.3914	4.3452	10.6215
20	0.1	1.1448	0.6005	1.2706
	1	2.6321	1.1937	2.7297
	10	5.5835	2.3917	5.7293
	100	11.9411	4.8535	12.2314
25	0.1	1.2647	0.6469	1.3791
	1	2.9007	1.2970	2.9932
	10	6.1679	2.6092	6.3482
	100	13.3229	5.3493	13.6324
30	0.1	1.3711	0.6886	1.4763
	1	3.1390	1.3887	3.2290
	10	6.6971	2.8109	6.9368
	100	14.5790	5.8683	14.9396

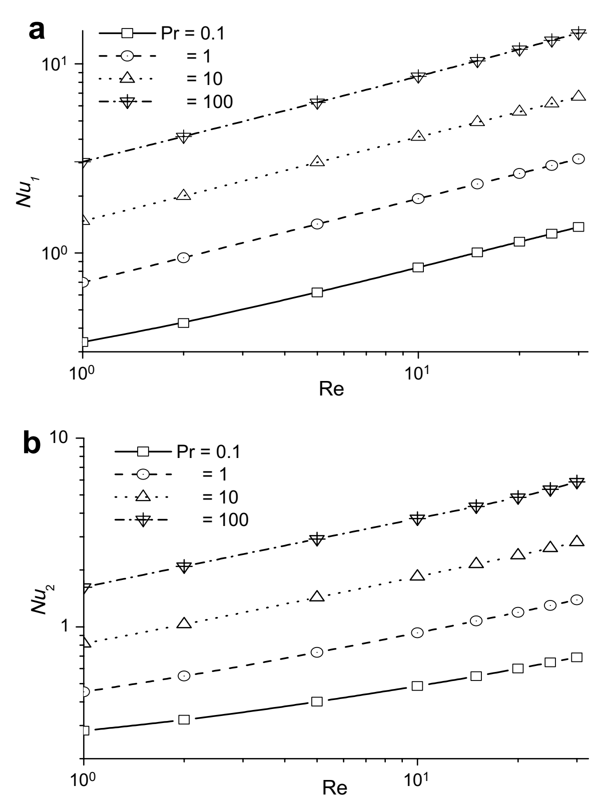

Fig. 1. Steady values of the Nu numbers function of Re for different Pr values: (a) upstream cylinder and (b) downstream cylinder.

where

$$A = \frac{\cosh\eta - \cos\xi}{\bar{c}}.$$

The appropriate boundary conditions are

- cylinder surfaces ($\eta = \eta_i,\ i = 1, 2$)

$$Z = Z_{c,i}, \quad i = 1, 2, \tag{4a}$$

- free stream ($\eta = \xi = 0$)

$$Z = 0, \tag{4b}$$

- symmetry axis ($\xi = 0$ and $\eta \neq 0,\ \xi = \pi$)

$$\frac{\partial Z}{\partial \xi} = 0. \tag{4c}$$

The dimensionless initial conditions are

$$\tau = 0, \quad Z_{c,i} = 1, \quad Z(\eta \neq \eta_i) = 0, \quad i = 1, 2. \tag{5}$$

The quantities of interest used to characterize the heat transfer are

- cylinder dimensionless temperature, $Z_{c,1}$ and $Z_{c,2}$;
- instantaneous local Nusselt number, $Nu_i(\xi)$, $i = 1, 2$;
- overall (surface average) instantaneous Nusselt number, Nu_i, $i = 1, 2$.

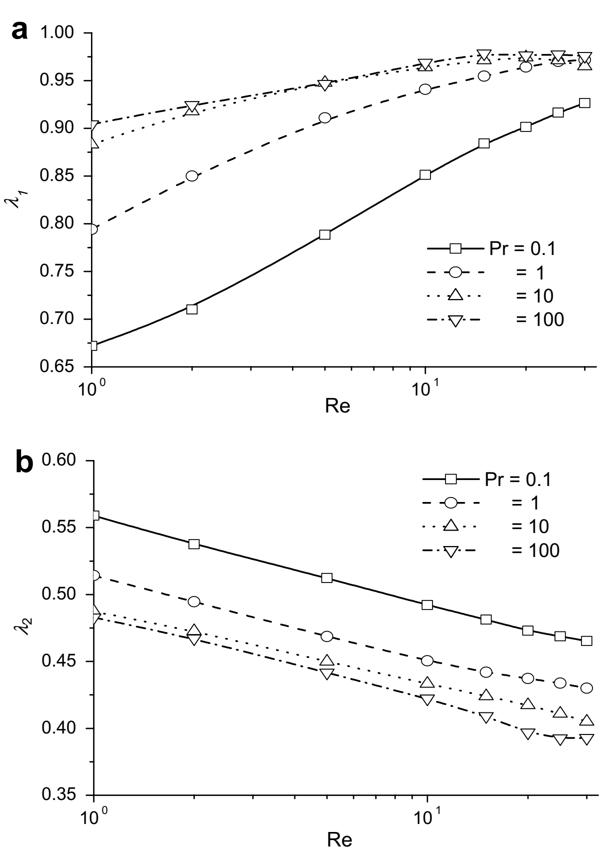

Fig. 2. The variation of the *Nu* numbers ratio with *Re* for different *Pr* values: (a) upstream cylinder and (b) downstream cylinder.

Considering as driving force the temperature difference $(T_{c,i} - T_\infty)$ and the diameters of the cylinders as characteristic length, $Nu_i(\xi)$ and Nu_i were calculated in bipolar cylindrical coordinates by the relations:

$$Nu_i(\xi) = -\frac{d_i}{d_1}\frac{1}{Z_{c,i}}\frac{\cosh\eta - \cos\xi}{\bar{c}}\left.\frac{\partial Z}{\partial \eta}\right|_{\eta=\eta_i}, \quad i = 1, 2, \tag{6a}$$

$$Nu_i = \frac{d_i}{d_1} Z_{c,i}^{-1} \frac{2}{\pi}\int_0^{\pi} \left.\frac{\partial Z}{\partial \eta}\right|_{\eta=\eta_i} \mathrm{d}\xi, \quad i = 1, 2. \tag{6b}$$

3. Method of solution

The values of the dimensionless stream function were calculated numerically. The numerical solution of the Navier–Stokes equations is presented in [1].

The energy balance equations were solved numerically. The mathematical model equations (3) is a system formed by a 2D parabolic partial differential equation (PDE) that describes the heat transfer in the fluid phase and two ordinary differential equations (ODEs) that describe the energy balance of the cylinders. The 2D domain $[\eta_1, \eta_2] \times [0, \pi]$ was transformed into the unit square. Eq. (3a) was discretized with the exponentially fitted scheme [11]. The discrete parabolic equation was solved by the implicit ADI method.

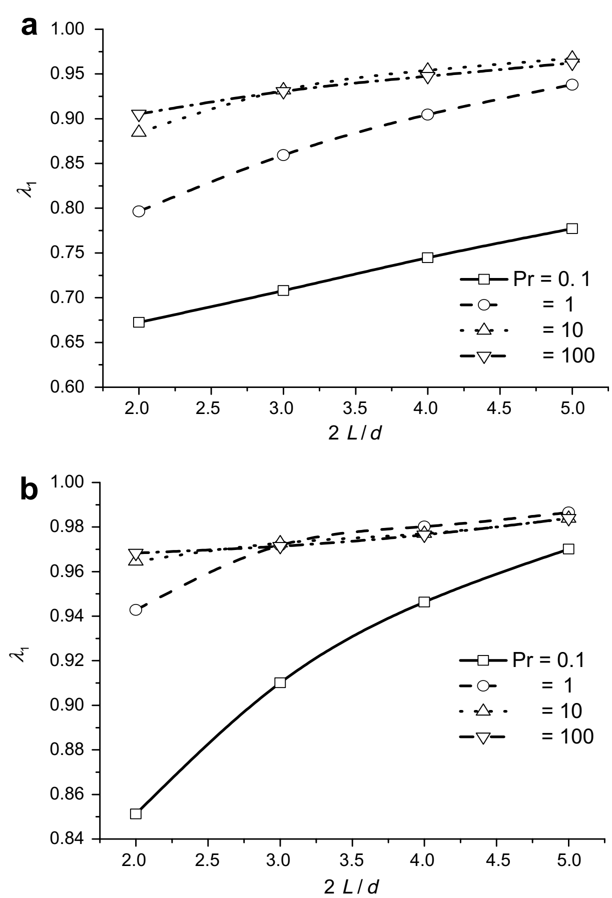

Fig. 3. The influence of the gap between cylinders on the *Nu* numbers ratio of the leading cylinder: (a) $Re = 1$ and (b) $Re = 10$.

Numerical experiments were made on spatial meshes with 65×65, 129×129, 257×257 and 513×513 points.

The ODEs were integrated by an explicit modified Euler algorithm. The integral from relations (3b) was calculated by the Newton–Simpson 3/8 rule. The time step was variable and changed from the start of the computation to the final stage. The initial and final values of the time step depend on the parameter values.

4. Results

The dimensionless groups of the present mathematical model are Re, Pr, Ξ, $2L_1/d_1$, and d_1/d_2. Numerical solutions of the Navier–Stokes equations were obtained in [1] for $1 \leqslant Re \leqslant 30$, $d_1 = d_2 = d$ and $2L/d \in [2,5]$ (for $d_1/d_2 = 0.5$ and 2, we obtained numerical solutions in [1] only for $2L_1/d_1 = 2$). Under these conditions, the values used in this work for Re, $2L/d$, and d_1/d_2 are $1 \leqslant Re \leqslant 30$, $2L/d \in [2,5]$, $d_1/d_2 = 1$. In all computations, the Prandtl number of the fluid phase, Pr, was considered equal to 0.1, 1, 10 and 100. Values of the volume heat capacity ratio, Ξ, between 0.01 and 100 cover the situations of practical interest. The results presented were obtained on a 513×513 mesh point.

Due to the lack of any data about the heat transfer from two cylinders in tandem at small Re numbers, we considered it useful to begin our investigation with the case of cylinders with constant temperature.

4.1. Cylinders with constant temperature

Table 1 summarizes our computations for $2L/d = 2$. The values calculated for the isolated cylinder are presented in the last column of Table 1. Figs. 1 and 2 show the influence of Re and Pr on Nu_1, Nu_2 (Fig. 1) and λ_1, λ_2 (Fig. 2). The influence of the gap between cylinders on the Nu numbers ratio is plotted in Figs. 3 and 4.

The following observations can be made from the data presented in Table 1 and Figs. 1–4:

- the values of the Nu number for both cylinders in tandem are smaller than the value of the Nu number for the isolated cylinder; the interaction effects are stronger for the downstream cylinder;
- for a given Pr number value, the increase in Re increases Nu for both cylinders; for a given Re, the increase in Pr increases Nu for both cylinders; the difference between λ_1 and λ_2, increases with the increase in Re and/or Pr;

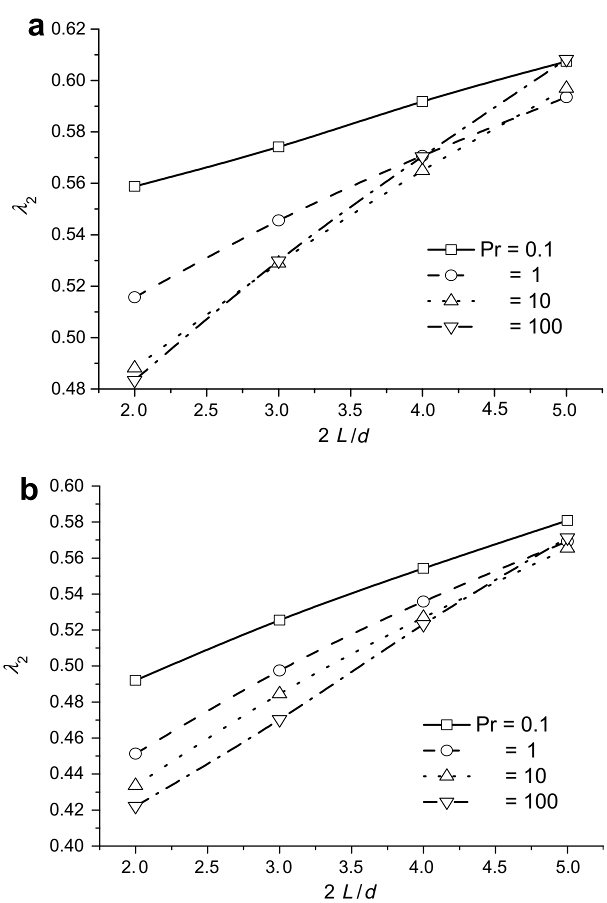

Fig. 4. The influence of the gap between cylinders on the Nu numbers ratio of the trailing cylinder: (a) $Re = 1$ and (b) $Re = 10$.

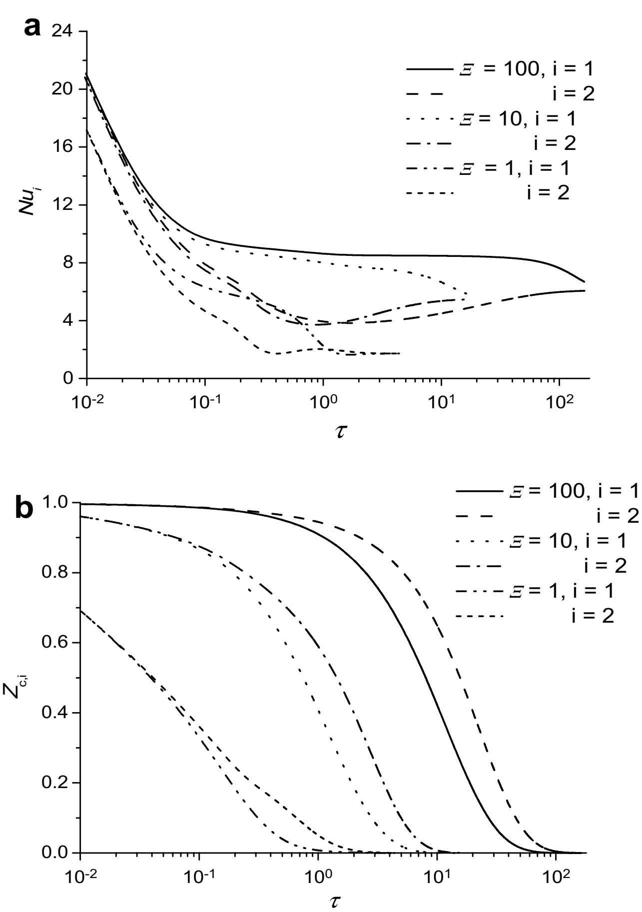

Fig. 5. Time variation of the average Nu numbers and dimensionless temperature of the cylinders for $Re = 10$, $Pr = 100$ and $\Xi \geqslant 1$.

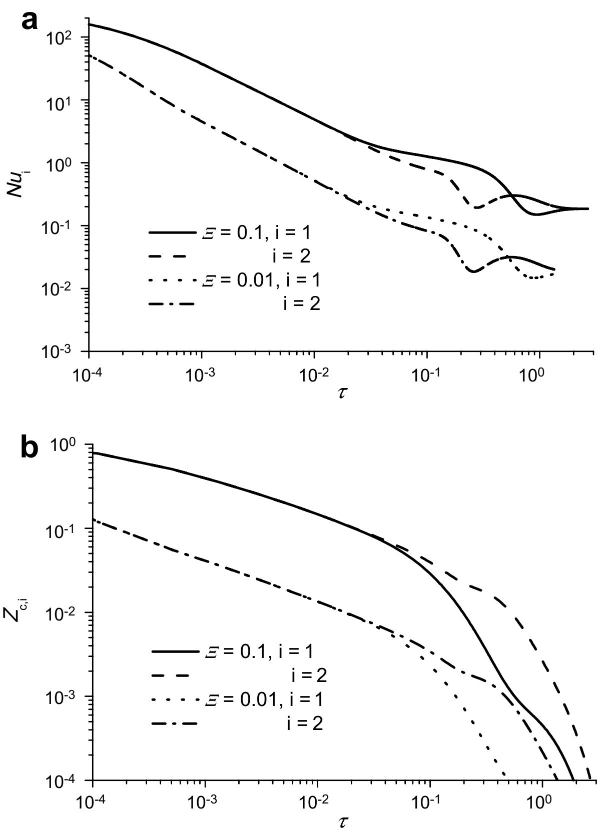

Fig. 6. Time variation of the average *Nu* numbers and dimensionless temperature of the cylinders for $Re = 10$, $Pr = 100$ and $\Xi < 1$.

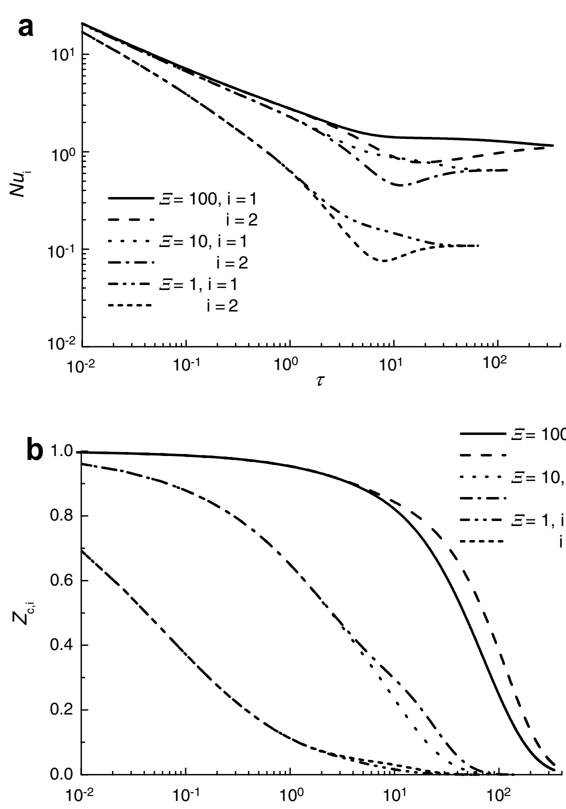

Fig. 7. Time variation of the average *Nu* numbers and dimensionless temperature of the cylinders for $Re = 1$, $Pr = 10$ and $\Xi \geqslant 1$.

- the increase in the distance between the two cylinders decreases the interaction effects.

Thus, we may conclude that the forced convection heat transfer between two in-line circular cylinders with constant temperature follows the *general rules* of the tandem interactions.

4.2. Cylinders with variable temperature

The key quantities that influence the thermal interaction between two spheres in tandem in creeping flow at moderate Peclet numbers are the gap between spheres and the volume heat capacity ratio [5]. As we mentioned previously, we could not obtain numerical solutions of the Navier–Stokes equations for small gaps between cylinders. For this reason, the present analysis is focused on the influence of *Re*, *Pr* and Ξ on the heat transfer rate for $2L/d \geqslant 2$.

We begin our analysis with the influence of *Re*, *Pr* and Ξ on the heat transfer rate for $2L/d = 2$ (the gap between cylinders is equal to the diameter of the cylinders). Four cases were selected for presentation: (a) $Re = 10$, $Pr = 100$ (high convection rate; convection is the dominant mechanism of transport – Figs. 5 and 6); (b) $Re = 1$, $Pr = 10$ (moderate to

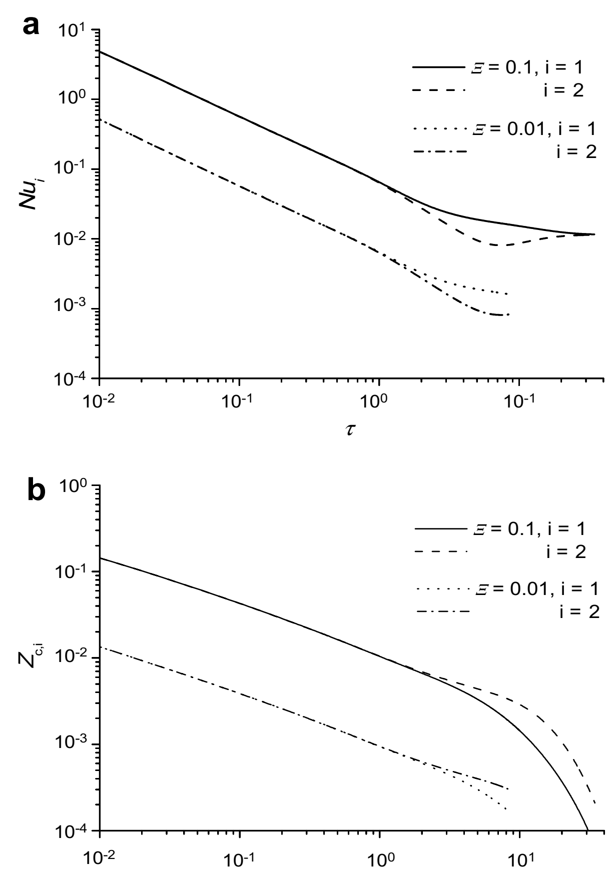

Fig. 8. Time variation of the average *Nu* numbers and dimensionless temperature of the cylinders for $Re = 1$, $Pr = 10$ and $\Xi < 1$.

low convection rate; convection is the dominant mechanism of transport – Figs. 7 and 8); (c) $Re = 1$, $Pr = 1$ (equal rates of convection and conduction – Figs. 9 and 10) and (d) $Re = 1$, $Pr = 0.1$ (conduction is the dominant mechanism of transport – Figs. 11 and 12). For each *Re* and *Pr* value, the values of the average *Nu* numbers and the dimensionless temperature of the cylinders are plotted for $\Xi = 100$, 10, 1, 0.1 and 0.01.

The time evolution of the average *Nu* numbers for cylinders with constant temperature has the same characteristics independent of *Re* and *Pr* values (we considered that it is not necessary to plot here this time evolution). At short times, the two *Nu* numbers coincide. For large times, the average *Nu* numbers separate and reach two distinct steady state values (see Table 1). The increase in *Re* and/or *Pr* increases the steady *Nu* values.

Figs. 5–12 show that the influence of convection on the thermal interaction between two cylinders with uniform temperature does not reduce only to the traditional aspects, i.e. the increase in *Re* and/or *Pr* increases the heat transfer rate. With regard to the *Re* and *Pr* values, the time evolution of the average *Nu* numbers is different. The main aspect that expresses this effect is the *Nu* numbers separation. For high to moderate values of the product *RePr*, the separation point occurs very clearly for all Ξ values.

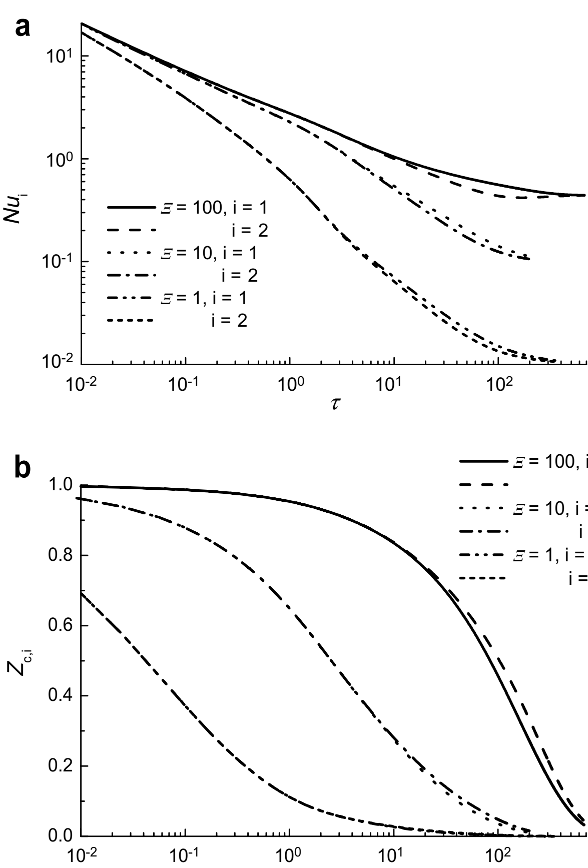

Fig. 9. Time variation of the average *Nu* numbers and dimensionless temperature of the cylinders for $Re = 1$, $Pr = 1$ and $\Xi \geqslant 1$.

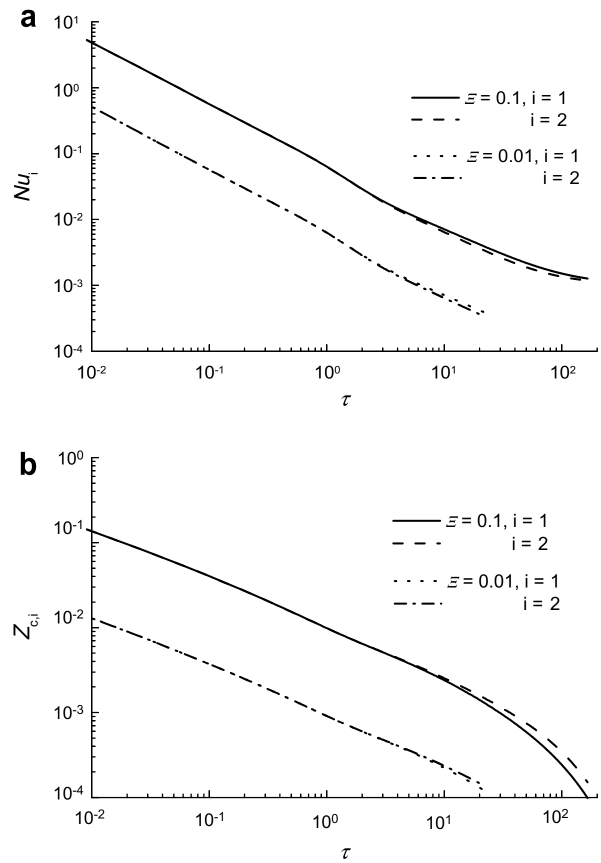

Fig. 10. Time variation of the average *Nu* numbers and dimensionless temperature of the cylinders for $Re = 1$, $Pr = 1$ and $\Xi < 1$.

The cylinders exhibit different average *Nu* values and have different dimensionless temperatures. It must be mentioned that for $\Xi = 0.01$, the separation of the *Nu* numbers occurs for $Z_{c,i} \approx 10^{-2}$. For $RePr = 1$, the separation of the *Nu* numbers is present but its influence on the heat transfer is less significant. When the thermal conduction is the dominant mechanism of transport, the *Nu* numbers and the dimensionless temperatures of the two cylinders are approximately equal. The relative difference between Nu_1 and Nu_2 does not exceed 3%. When $\tau \to \infty$, $Nu_i \to 0$.

Another aspect that seems to be important is the *convergence* of the two *Nu* numbers at large times. This aspect can be easily explained. After the separation point, Nu_1 is greater than Nu_2 and the dimensionless temperature of the leading cylinder decreases faster than the dimensionless temperature of the trailing cylinder. When the difference between $Z_{c,1}$ and $Z_{c,2}$ becomes significant, i.e. $Z_{c,1} \approx 10^{-1} Z_{c,2}$, the impact of the leading cylinder on the trailing cylinder decreases significantly and the heat transfer rate of the trailing cylinder increases. At high convection rates this process is very fast. The heat transfer rate of the trailing cylinder may reach values greater than those of the leading cylinder, $Nu_2 > Nu_1$, the reverse process begins and so on. It must be mentioned that these phenomena take place at very small values of $Z_{c,i}$, $Z_{c,i} \leqslant 10^{-3}$.

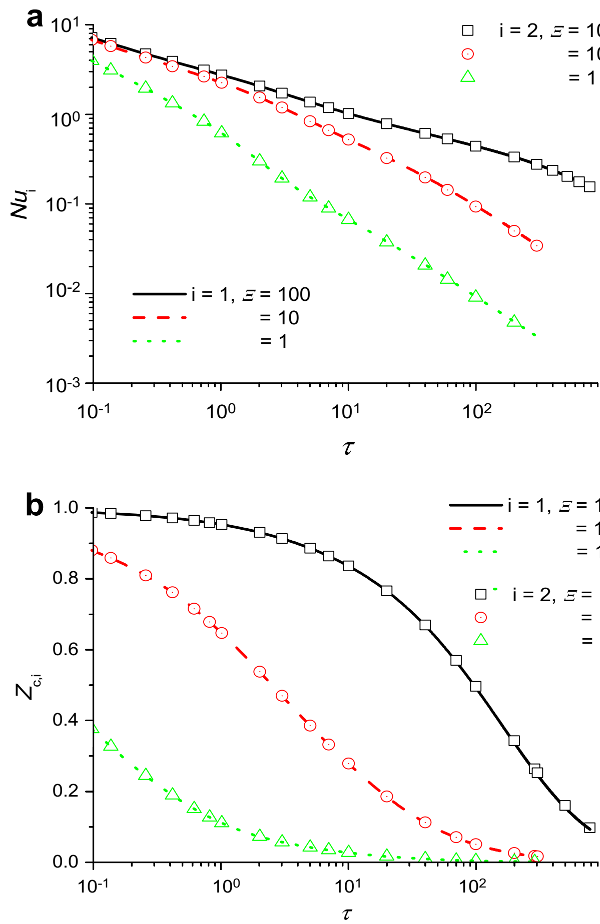

Fig. 11. Time variation of the average Nu numbers and dimensionless temperature of the cylinders for $Re = 1$, $Pr = 0.1$ and $\Xi \geqslant 1$.

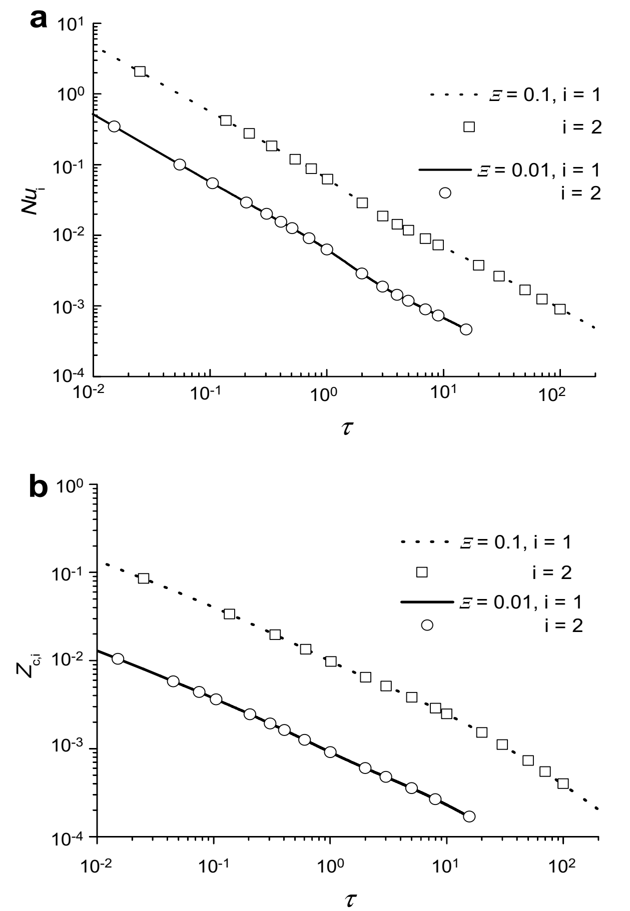

Fig. 12. Time variation of the average Nu numbers and dimensionless temperature of the cylinders for $Re = 1$, $Pr = 0.1$ and $\Xi < 1$.

Concerning the numerical simulations made at $2L/d = 2$, the following aspects should be also mentioned:

- for given Re and Ξ values, the increase in Pr increases the heat transfer rate;
- for given Pr and Ξ values, the increase in Re increases the heat transfer rate;
- for given Re and Pr values, the increase in Ξ increases the heat transfer rate;
- negative values of the average Nu numbers were not obtained;
- the asymptotic values of the average Nu numbers are smaller than the asymptotic average Nu number of the isolated cylinder.

The influence of the distance between cylinders on the heat transfer rate is presented in Fig. 13 ($Re = 10$, $Pr = 100$) and Fig. 14 ($Re = 1$, $Pr = 0.1$). We thought that these two cases express the salient features of the process. Two Ξ values were considered necessary for $Re = 10$ and $Pr = 100$, $\Xi = 100$ and $\Xi = 1$.

Fig. 13 shows that when convection is the dominant mechanism of transport, the increase in the gap between cylinders does not increase the average Nu numbers. This aspect is less evident for $\Xi = 100$ and very clear at $\Xi = 1$. The discontinuities in Nu_2 in Fig. 13b are due to the fact that Nu_2 takes negative values. The situation depicted in Fig. 13 is similar to that presented in [5]. Fig. 14 shows that when conduction is the dominant mechanism of transport, the gap between cylinders does not influence significantly Nu_i.

We think that the interaction mechanism presented in [5] remains valid for the present problem. The differences between the hydrodynamic regimes (Stokes flow in [5] and low Re numbers flow here) influence the heat transfer but do not change the salient features of the interaction. For this reason, we consider that it is not necessary to repeat here all the facts discussed in [5].

The heat transfer rate of the tandem cylinders with uniform temperature depends on the values of the cylinders' dimensionless temperature and temperature gradients. High heat transfer rates are obtained when the interaction takes place at high values of the cylinders' dimensionless temperature (i.e. values closed to one) and temperature gradients. The values of the cylinders' dimensionless temperature and temperature gradients during the interaction depend on Re, Pr, Ξ and the gap between cylinders. The

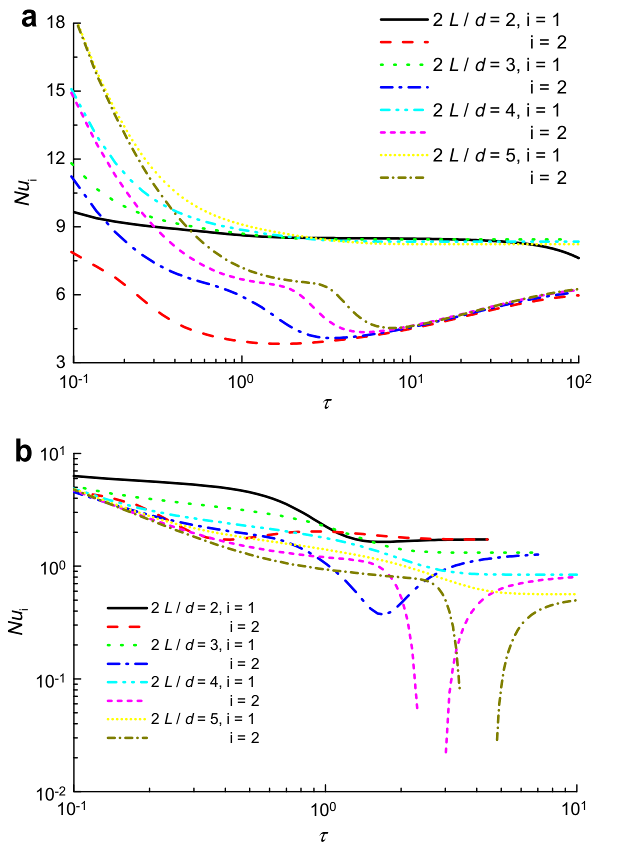

Fig. 13. The influence of the gap between cylinders on the average Nu numbers for $Re = 10$ and $Pr = 100$: (a) $\Xi = 100$ and (b) $\Xi = 1$.

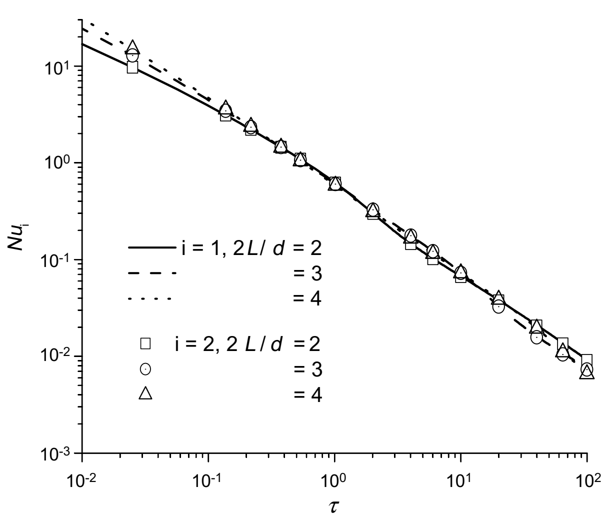

Fig. 14. The influence of the gap between cylinders on the average Nu numbers for $Re = 1$, $Pr = 0.1$ and $\Xi = 1$.

interaction occurs and develops at high values of the cylinders' dimensionless temperature and temperature gradients for high values of volume heat capacity ratio, small gaps between cylinders and high convection rates. Two supplementary aspects should be taken into consideration in order to explain the behaviour of the system for $Re\,Pr \leqslant 1$:

(i) for low and very low convection rates, even for cylinders with constant temperature (see Table 1), the difference between Nu_1 and Nu_2 is not high (note that for $Re = 0$, $Nu_1 = Nu_2$);
(ii) in conjugate heat transfer, the average Nu number tends to zero when Peclet number tends to zero.

5. Conclusions

The objective pursued in this work was to obtain a better understanding of unsteady heat transfer from tandem cylinders in low Reynolds numbers flow. The values assumed for the Prandtl number of the fluid phase are 0.1, 1, 10 and 100. The analysis was directed toward the influence of the product $Re\,Pr$ on the heat transfer rate at different values of volume heat capacity ratio. The cylinders have the same diameter and identical physical properties. The gap between cylinders was considered equal or greater than the diameter of the cylinder.

The numerical results presented in the previous section show that the heat transfer from tandem cylinders with uniform temperature has its own specific rules. In almost all situations the average Nu numbers do not reach a frozen asymptotic value. High heat transfer rates were obtained when the interaction begins and develops at high values of the cylinders' dimensionless temperature. High convection rate, small gaps between cylinders and high values of the volume heat capacity ratio lead to high heat transfer rates. The influence of convection rate on the heat transfer of tandem cylinders is the most important result obtained in this work. The evolution of the system for $Re\,Pr > 1$ (convection dominates conduction) is completely different in comparison with $Re\,Pr < 1$ (conduction dominates convection).

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