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Solar cell array parameters using Lambert W-function

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Abstract

Exact closed-form solution based on Lambert W-function are presented to express the transcendental current–voltage characteristic containing parasitic power consuming parameters like series and shunt resistances for solar cell array. Maple software was used to solve the transcendental equation of solar cell array.

Keywords: Lambert W-function; Solar cell array

1. Introduction

In a solar cell array, cells are connected in parallel and series to provide required terminal voltage and current ratings. Ideally, these cells have identical electrical characteristics when illuminated but the practical case is not so. Due to the relatively high cost of a solar cell array, the main objective of the system designer is to extract maximum available electrical power output at all insolation levels, for maximum utilization efficiency of the system.

There is a vast application of photovoltaic power in the remote rural areas of developing countries [1]. The most successful application is to use a solar array to power a dedicated load such as a DC-motor [2–6]. The key to their success is simplicity, for example, direct coupling, no DC–AC conversion, no storage batteries.

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This arrangement is typically used on non-identical loads such as water pumps, which need not operate continuously, and water can be used directly or stored easily. In field conditions the cellular array circuits exhibit faults arising due to mismatch losses such as the net array output power is less than the sum of output power of constituent solar cells. The mismatch loss tends to enhance with time due to degradation resulting from aging of cells. The mismatch loss can be reduced if each row of parallel string is shunted by a bypass diode.

Efficiency of an array, which is affected by electrical mismatches, can also be enhanced by such redundant circuit design as series parallel array [7]. In this scheme a circuit is divided into series blocks. One or more of these series blocks can be bridged by bypass diode.

Solar cells in array are never identical, which complicates the analysis of a large photovoltaic array operation under different load and environmental conditions. So efforts have been made to combine the cell parameters of the array into a single aggregate model to simplify calculations.

In earlier works the array parameters were derived by different methods such as (a) incomplete analytical methods, (b) complete analytical methods, (c) simulation methods [8–11]. Some work deals with calculating series and shunt resistance of array with many approximations [12].

The present work deals with calculation of various array parameters using Lambert W-function method to solve the current–voltage relationship of a solar cell [13–15]. The significance of this method is that it uses no approximations and provides exact mathematical relations for various array parameters.

2. Theory

The current–voltage relation of single solar cell in a photovoltaic array is given by

$$\ln\left(\frac{i+I_{\mathrm{ph}}}{\mathrm{I_o}}-\frac{V-iR_{\mathrm{s}}}{I_{\mathrm{o}}R_{\mathrm{sh}}}+1\right)=\frac{V-iR_{\mathrm{s}}}{nV_{\mathrm{th}}}, \tag{1}$$

where V and i are terminal voltage and current, respectively; I_{ph} is photocurrent; I_{o} is the diode reverse saturation current; R_{s} and R_{sh} are series and shunt resistance, respectively; n is diode ideality factor; V_{th} is the thermal voltage.

In a series array consisting of N identical cells where the current through the array is equal to the current through individual cell it can be shown that

$$I_{\mathrm{ph_a}}=I_{\mathrm{ph}},\quad I_{\mathrm{o_a}}=I_{\mathrm{o}},\quad R_{\mathrm{s_a}}=NR_{\mathrm{s}},\quad R_{\mathrm{sh_a}}=NR_{\mathrm{sh}},\quad V_{\mathrm{th_a}}=NV_{\mathrm{th}}.$$

And for parallel array

$$I_{\mathrm{ph_a}}=NI_{\mathrm{ph}},\quad I_{\mathrm{o_a}}=NI_{\mathrm{o}},\quad R_{\mathrm{s_a}}=R_{\mathrm{s}}/N,\quad R_{\mathrm{sh_a}}=R_{\mathrm{sh}}/N,\quad V_{\mathrm{th_a}}=V_{\mathrm{th}}.$$

The explicit voltage equations for nth cell in an array as determined by solving Eq. (1) is

$$V_n=V_{\mathrm{oc}_n}+R_{\mathrm{sh}_n i_n}+R_{\mathrm{s}_n i_n}, \tag{2}$$

where V_{oc_n} is given by

$$V_{\mathrm{oc}_n} = -\text{Lambert W}\left(\frac{I_{\mathrm{o}_n} R_{\mathrm{sh}_n} \mathrm{e}^{(I_{\mathrm{ph}_n} R_{\mathrm{sh}_n}/AV_{\mathrm{th}_n})}}{AV_{\mathrm{th}_n}}\right) nV_{\mathrm{th}_n} + R_{\mathrm{sh}_n} I_{\mathrm{ph}_n}. \quad (3)$$

Similarly, the *I–V* equation of the array of the same form as for the single cell in terms of voltage is

$$V_{\mathrm{a}} = V_{\mathrm{oc_a}} + (R_{\mathrm{s_a}} + R_{\mathrm{sh_a}}) i_{\mathrm{a}}, \quad (4)$$

where $V_{\mathrm{oc_a}}$ is given by

$$V_{\mathrm{oc_a}} = -\text{Lambert W}\left(\frac{I_{\mathrm{o_a}} R_{\mathrm{sh_a}} \mathrm{e}^{(I_{\mathrm{ph_a}} R_{\mathrm{sh_a}}/AV_{\mathrm{th_a}})}}{AV_{\mathrm{th_a}}}\right) nV_{\mathrm{th_a}} + R_{\mathrm{sh_a}} I_{\mathrm{ph_a}}. \quad (5)$$

Two different photocurrents are introduced for array: $I_{\mathrm{pho_a}}$ and $I_{\mathrm{ph_a}}$, $I_{\mathrm{pho_a}}$ is the photocurrent at open circuit voltage and current independent whereas $I_{\mathrm{ph_a}}$ is the array photocurrent at load and current dependent, where $I_{\mathrm{pho_a}}$ is given by

$$I_{\mathrm{pho_a}} = \frac{V_{\mathrm{oc_a}} + I_{\mathrm{o_a}} R_{\mathrm{sh_a}} \mathrm{e}^{(V_{\mathrm{oc_a}}/AV_{\mathrm{th_a}})}}{\mathrm{R_{sh_a}}}.$$

The explicit current equations for the *n*th cell in an array as determined by solving Eq. (1) are

$$i_n = I_{\mathrm{sc}_n} + \frac{V_n}{R_{\mathrm{s}_n} + R_{\mathrm{sh}_n}}, \quad (6)$$

where I_{sc_n} (short circuit current for the *n*th cell of the array) is

$$I_{\mathrm{sc}_n} = -\frac{\left(-\text{Lambert W}\left(\frac{R_{\mathrm{s}_n} I_{\mathrm{o}_n} R_{\mathrm{sh}_n} \mathrm{e}^{(R_{\mathrm{sh}_n} R_{\mathrm{s}_n} I_{\mathrm{ph}_n}/AV_{\mathrm{th}_n}(R_{\mathrm{s}_n}+R_{\mathrm{sh}_n}))}}{R_{\mathrm{s}_n} AV_{\mathrm{th}_n} + R_{\mathrm{sh}_n} AV_{\mathrm{th}_n}}\right) + \frac{R_{\mathrm{sh}_n} R_{\mathrm{s}_n} I_{\mathrm{ph}_n}}{AV_{\mathrm{th}_n}(R_{\mathrm{s}_n} + R_{\mathrm{sh}_n})}\right) AV_{\mathrm{th}_n}}{R_{\mathrm{s}_n}}. \quad (7)$$

Expression for array current is

$$i_{\mathrm{a}} = I_{\mathrm{sc_a}} + \frac{V_{\mathrm{a}}}{R_{\mathrm{s_a}} + R_{\mathrm{sh_a}}}. \quad (8)$$

3. Series array

When cells are connected in series, current through the array is equal to minimum current through any cell while voltage across the array is equal to the sum of voltages across individual cells. Hence, the voltage relation across the array will be

$$V_{\mathrm{a}}(i_{\mathrm{a}}) = \sum_{n=1}^{N} V_n(i_{\mathrm{a}}) \quad (9)$$

substituting V_n and i_{a} in Eq. (9) different array parameters can be determined.

We have

$$V_{\mathrm{th_a}} = \sum_{n=1}^{N} V_{\mathrm{th}_n}. \quad (10)$$

for identical cells

$$V_{\mathrm{th_a}} = N V_{\mathrm{th}_n}. \quad (11)$$

Series resistance in array

$$R_{\mathrm{s_a}} = \sum_{n=1}^{N} R_{\mathrm{s}_n}. \quad (12)$$

Shunt resistance in array

$$R_{\mathrm{sh_a}} = \sum_{n=1}^{N} R_{\mathrm{sh_n}}. \quad (13)$$

Reverse saturation current

$$I_{\mathrm{o_a}} = \mathrm{surd}\left(\prod_{n=1}^{N} I_{\mathrm{o}_n}^{(1/\eta_n)}, N\right), \quad (14)$$

where

$$I_{\mathrm{o}_n} = \frac{R_{\mathrm{sh}_n} i_{n_n} + R_{\mathrm{sh}_n} I_{\mathrm{ph}_n} - V_n + i_{n_n} R_{\mathrm{s}_n}}{\mathrm{e}^{\left(-(-V_n + i_{n_n} R_{\mathrm{s}n})/nV_{\mathrm{th}n}\right)} R_{\mathrm{sh}_n}}$$

and

$$\eta_n = \frac{V_{\mathrm{th_a}}}{N V_{\mathrm{th}_n}}.$$

Photocurrent for open circuit array

$$I_{\mathrm{pho_a}} = \mathrm{surd}\left(\prod_{n=1}^{N} I_{\mathrm{ph}_n}^{(1/\eta_n)}, N\right), \quad (15)$$

where

$$I_{\mathrm{ph}_n} = \frac{-R_{\mathrm{sh}_n} i_n + R_{\mathrm{sh}_n} I_{\mathrm{o}_n} \mathrm{e}^{((V_n - i_n R_{\mathrm{s}n})/V_{\mathrm{th}n} A)} + V_n - i_n R_{\mathrm{s}_n}}{R_{\mathrm{sh}_n}}.$$

Photocurrent for array

$$I_{\mathrm{ph_a}} = \frac{I_\mathrm{a}}{1 - \mathrm{surd}\left(\prod_{n=1}^{N}\left(1 - (I_\mathrm{a}/I_{\mathrm{ph}_n})^{(1/\eta_n)}\right), N\right)}, \quad (16)$$

where surd(a,N) implies Nth root of a.

It can be concluded from above equations that for series array having identical cells and so photocurrents, $I_{\text{pho}_\text{a}} = I_{\text{ph}_\text{a}} = I_{\text{ph}}$.

4. Parallel array

When cells are connected in parallel, current through the array is equal to sum of currents through individual cells while voltage across the array is equal to voltage across any individual cell.

$$V_\text{a} = V_n \tag{17}$$

and

$$i_\text{a}(V) = \sum_{n=1}^{N} i_n(V) \tag{18}$$

substituting $i_n(V)$ in the above equation and solving it we obtain

Series resistance for the array

$$R_{\text{s}_\text{a}} = \frac{1}{\sum_{n=1}^{N} R_{\text{s}_n}}. \tag{19}$$

Shunt resistance for the array

$$R_{\text{sh}_\text{a}} = \frac{1}{\sum_{n=1}^{N} (1/R_{\text{sh}_n})} \tag{20}$$

$$\frac{V_{\text{th}_\text{a}} \ln(I_{\text{o}_\text{a}})}{R_{\text{s}_\text{a}}} = \sum_{n=1}^{N} \frac{V_{\text{th}_n} \ln(I_{\text{o}_n})}{R_{\text{s}_n}}.$$

5. Maximum power

A simple exact expression for calculating maximum power of an array could not be obtained due to constraints of maple software, a graphical representation of array power versus array current and array voltage is made in Figs. (1) and (2). The graphs are for an typical array with parameters $V_{\text{th}_\text{a}} = 0.73\,\text{V}$, $\text{I}_{\text{ph}} = 0.8\,\text{A}$ (at an insolation of $1000\,\text{W/m}^2$), $I_\text{o} = 0.5\,\text{mA}$, $R_\text{s} = 0.05\,\Omega$, $R_{\text{sh}} = 10^5\,\Omega$. The array consists of 18 parallel strings with 324 cells in series per string [16]. It is found that maximum power is transferred at certain value of array voltage and array current.

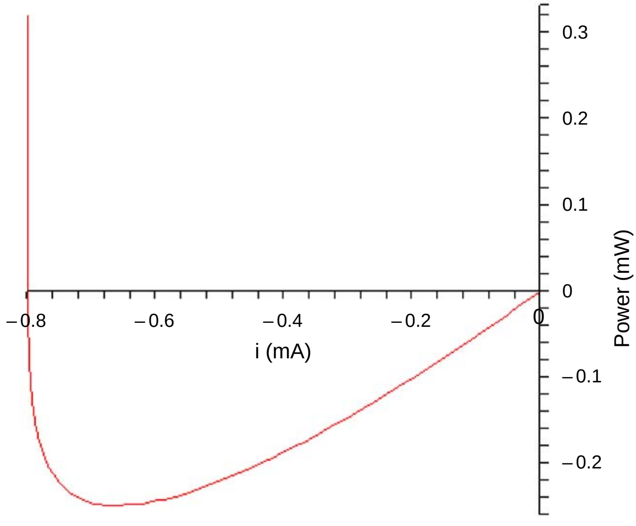

Fig. 1. Mixed array power vs. mixed array current.

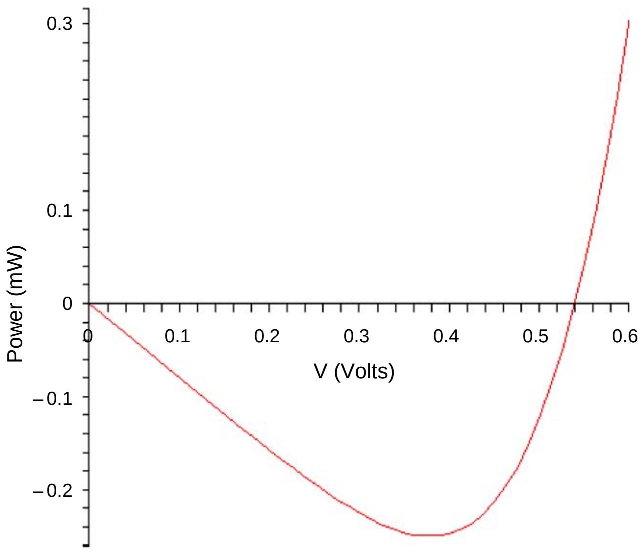

Fig. 2. Mixed array power vs. mixed array voltage.

6. Conclusion

In this paper, exact closed form solution based on Lambert W-function for array voltage and current are presented. Using these expressions various parameters for series and parallel array are obtained. Study for maximum power was done using graphical means.

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