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The discrete-time $Geo/Geo/1$ queue with negative customers and disasters

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Abstract

The arrival of a negative customer to a queueing system causes one ordinary customer to be removed (or killed) if any is present. The arrival of a disaster, on the other hand, kills all the customers in the system if any. In this paper, we extend the queueing theory on negative arrivals and disasters to the discrete-time $Geo/Geo/1$ queueing system. Specifically, we find the ergodicity condition and give explicit formulae for the stationary distribution. We also provide some numerical results to illustrate the effect of the parameters on several performance characteristics.

Keywords: Discrete-time model; Markov chain; Ergodicity; Negative customers; Disasters

1. Introduction

Recently there has been a rapid increase in the literature on queueing systems with negative arrivals. In its simplest version, a negative customer removes an ordinary (positive) customer according to some strategy. Therefore, a negative arrival represents some kind of work canceling signal. Since their introduction by Gelenbe [1], negative arrivals have been interpreted as inhibitor and synchronization signals in neural and queueing networks modelling. Intuitively, the introduction of negative arrivals makes the system less congested than if they were not present, so the existence of negative customers provides a mechanism to control an excessive congestion at the system. Also, negative arrivals can represent commands to delete some transaction, as in distributed computer systems or databases in which certain operations become impossible because of locking of data or inconsistency.

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Typical killing strategies considered are RCH (removal of customer at the head) and RCE (removal of customer at the end): RCH kills the customer in service and RCE kills the customer who arrived most recently regardless of whether he is in service or waiting, if any. A different kind of negative arrivals, called disasters, was introduced by Jain and Sigman [2]. A disaster (DST) kills simultaneously all the customers (all the work) in the system. DST has no effect to an empty system. DST is also called *mass exodus* [3] or *queue flushing* [4]. In this paper, we consider the $Geo/Geo/1$ queue having RCH, RCE and DST. However, in our case RCH is equivalent to RCE due to the memoryless property of the geometrical law.

It should be pointed out that queues with disasters could be considered as the basic models of computer systems in the presence of a virus or a reset order. In computer systems or networks, if a job (or a station) is infected, this job may transmit a virus when it is transferred to other processors (CPU, I/O, diskettes, etc). In this case, disasters have a role of a clearing operation of all stored messages present in the system.

There exist another queueing models where the customers can leave the system before completing his service (but not due to external causes as in this paper), they are called queueing systems with impatient or non-persistent customers.

On the other hand, there is a growing interest in the analysis of discrete-time queues due to their applications in communication systems and other related areas. The study of discrete-time queues was iniciated by Meisling [5] but its importance has been realized only in the last decade. In recent years, a number of queueing models have been analysed in discrete-time, details of which may be found in [6–8]. An investigation of the discrete-time queueing systems is important due to their applications to slotted systems such as slotted Aloha and ATM in BISDN. The machine cycle time of a processor and the transmission time of one cell in ATM are the elementary units of time in the slotted systems.

In the literature, both negative arrivals and disasters were applied to single-server queues [2,4,9,10], retrial queues [11,12], queueing networks and birth and death processes; nevertheless its study has been only focused on the continuous-time until now, but in this paper we extend this topic to the discrete-time systems.

The rest of the paper is organized as follows. In the next section, we give the mathematical description of the considered queueing system. In Sections 3 and 4, we analyse the discrete-time $Geo/Geo/1$ queueing system having both negative customers and disasters. Finally, in Section 5, we present some numerical results to illustrate the effect of the parameters on several performance characteristics.

2. The mathematical model

We consider a discrete-time queueing system where the time axis is segmented into a sequence of equal time intervals (called slots). It is assumed that all queueing activities (arrivals and departures) occur at the slot boundaries, and therefore they may occur at the same time. For mathematical clarity, we will suppose that the departures occur at the moment immediately before the slot boundaries and the arrivals occur at the moment immediately after the slot boundaries.

Customers arrive according to a geometric arrival process with rate p, that is, p is the probability that an arrival occurs in a slot. If, upon arrival, the server is idle, the service of the arriving customer commences immediately. Otherwise, the arriving customer either with probability q^+ joins the waiting line in order to be served (positive customer), or with complementary probability q^- kills the customer in service (negative customer) or all the customers in the system simultaneously (disaster). In the following two sections we will analyse both cases separately: negative customers and disasters.

It is always assumed that services can be started only at slot boundaries and their durations are integral multiples of a slot duration. Service times are independent and geometrically distributed with probability $\bar{s} = 1 - s$, where s is the probability that a customer does not finish his service in a slot.

We will suppose $0 < q^+ < 1$ and, in order to avoid trivial cases, $0 < p < 1$ and $0 < s < 1$.

At time m^+ (the instant immediately after time slot m), the system can be described by the process X_m, which denotes the number of customers in the system (including the one in service if any).

It can be easily shown that $\{X_m, \ m \in \mathbb{N}\}$ is the one-dimensional Markov chain of our queueing system, whose states space is $\{0, 1, 2, \ldots\}$.

Our first objective will be to find the stationary distribution

$$\pi_k = \lim_{m \to \infty} P[X_m = k], \ k \geqslant 0$$

of the Markov chain $\{X_m, \ m \in \mathbb{N}\}$. We introduce the auxiliary generating function

$$\varphi(z) = \sum_{k=1}^{\infty} \pi_k z^{k-1}, \ |z| \leqslant 1$$

in order to solve the Kolmogorov equations for the distribution π_k. It should be pointed out that $\varphi(z)$ is the marginal generating function of the number of customers in the waiting line when the server is busy.

3. The RCH killing discipline

In this section we consider the RCH (removal of customers from the head of the queue) discipline. This is appropriate for modelling server breakdowns where a customer in service is lost.

As we have said in the introduction, in our case RCH is equivalent to RCE due to the lack of memory of the geometrical law.

The one-step transition probabilities $p_{k',k} = P[X_{m+1} = k | X_m = k']$ are given by the formulae

$$p_{0,0} = \bar{p},$$

$$p_{1,0} = \bar{s}\bar{p} + spq^-,$$

$$p_{2,0} = \bar{s}pq^-,$$

$$p_{0,1} = p,$$

$$p_{1,1} = \bar{s}p + s\bar{p},$$

$$p_{2,1} = \bar{s}\bar{p} + spq^-,$$

$p_{3,1} = \bar{s}pq^-,$

$p_{k-1,k} = spq^+, k \geqslant 2,$

$p_{k,k} = \bar{s}pq^+ + s\bar{p}, k \geqslant 2,$

$p_{k+1,k} = \bar{s}\bar{p} + spq^-, k \geqslant 2,$

$p_{k+2,k} = \bar{s}pq^-, k \geqslant 2,$

where $\bar{p} = 1 - p$.

The Kolmogorov equations for the distribution π_k are

$$\pi_0 = \bar{p}\pi_0 + (\bar{s}\bar{p} + spq^-)\pi_1 + \bar{s}pq^-\pi_2, \tag{1}$$

$$\pi_1 = p\pi_0 + (\bar{s}p + s\bar{p})\pi_1 + (\bar{s}\bar{p} + spq^-)\pi_2 + \bar{s}pq^-\pi_3, \tag{2}$$

$$\pi_k = spq^+\pi_{k-1} + (\bar{s}pq^+ + s\bar{p})\pi_k + (\bar{s}\bar{p} + spq^-)\pi_{k+1} + \bar{s}pq^-\pi_{k+2}, \ k \geqslant 2 \tag{3}$$

and the normalization condition is $\sum_{k=0}^{\infty} \pi_k = 1$.

Multiplying (3) by z^k and summing over k leads to

$$\left[(\bar{s} + sz)\left(\frac{pq^-}{z} + \bar{p}\right) + \bar{s}pq^+z - z\right]\pi_1 + [(\bar{s} + sz)pq^- + \bar{s}\bar{p}z]\pi_2$$
$$+ \bar{s}pq^-z\pi_3 = \left[(\bar{s} + sz)\left(\frac{pq^-}{z} + \bar{p} + pq^+z\right) - z\right]\varphi(z). \tag{4}$$

By substituting (1)–(2) into (4), we have

$$\left[(\bar{s} + sz)\left(\frac{pq^-}{z} + \bar{p} + pq^+z\right) - z\right]\varphi(z) = p(1 - z)\left[\pi_0 + \bar{s}q^-\frac{z+1}{z}\pi_1\right],$$

which can be written as

$$[s\rho q^+z^2 - (\bar{p} + \rho q^-)z - pq^-]\varphi(z) = -\rho[z\pi_0 + \bar{s}q^-(z + 1)\pi_1], \tag{5}$$

where $\rho = p/\bar{s}$ is the load of the system.

Note that the polynomial in the left-hand side of Eq. (5) has two roots z_1^* and z_2^*, which satisfy the conditions $-1 < z_1^* < 0$ and $z_2^* > 0$.

Setting $z = z_1^*$ in (5), we get

$$z_1^*\pi_0 + \bar{s}q^-(z_1^* + 1)\pi_1 = 0. \tag{6}$$

Taking into account the expression

$$\varphi(1) = \lim_{z\to 1} \frac{-\rho[z\pi_0 + \bar{s}q^-(z + 1)\pi_1]}{s\rho q^+z^2 - (\bar{p} + \rho q^-)z - pq^-} = \frac{\pi_0 + 2\bar{s}q^-\pi_1}{1 - \rho(q^+ - q^-)}\rho$$

and the normalization condition $\pi_0 + \varphi(1) = 1$, we obtain

$$(1 + 2\rho q^-)\pi_0 + 2pq^-\pi_1 = 1 - \rho(q^+ - q^-). \tag{7}$$

The system of Eqs. (6)–(7) has a unique solution, since the determinant

$$\begin{vmatrix} z_1^* & \bar{s}q^-(z_1^* + 1) \\ 1 + 2\rho q^- & 2pq^- \end{vmatrix} = -q^-[\bar{s}(z_1^* + 1) + 2p(q^- - q^+ z_1^*)]$$

is not equal to zero. This solution is given by

$$\pi_0 = \frac{1 - \rho(q^+ - q^-)}{(1 - 2\rho q^+)z_1^* + 2\rho q^- + 1}(z_1^* + 1)$$

$$\pi_1 = \frac{1 - \rho(q^+ - q^-)}{(1 - 2\rho q^+)z_1^* + 2\rho q^- + 1}\frac{-z_1^*}{\bar{s}q^-}.$$

From the expressions for π_0 and π_1, we conclude that $\rho(q^+ - q^-) < 1$ is a necessary condition for the ergodicity of the Markov chain.

Lemma 1. *The condition $z_2^* > 1$ is necessary and sufficient for the stability of the system.*

Proof. If $0 < z_2^* \leqslant 1$, then choosing $z = z_2^*$ in (5), we obtain

$$z_2^*\pi_0 + \bar{s}q^-(z_2^* + 1)\pi_1 = 0. \tag{8}$$

The system of Eqs. (6) and (8) has the solution $\pi_0 = 0 = \pi_1$ and therefore the Kolmogorov equations do not have a nontrivial solution.

If $z_2^* > 1$, then the system is obviously stable. □

It is readily seen that

$$z_2^* > 1 \Leftrightarrow \rho(q^+ - q^-) < 1$$

and consequently $\rho(q^+ - q^-) < 1$ is the ergodicity condition of the Markov chain.

Turning to Eq. (5) and considering that

$$z\pi_0 + \bar{s}q^-(z + 1)\pi_1 = \frac{1 - \rho(q^+ - q^-)}{(1 - 2\rho q^+)z_1^* + 2\rho q^- + 1}(z - z_1^*),$$

we have

$$\varphi(z) = \frac{1 - \rho(q^+ - q^-)}{(1 - 2\rho q^+)z_1^* + 2\rho q^- + 1}\frac{1}{sq^+(z_2^* - z)}.$$

Let us observe that the probability generating function of the number of customers in the system (i.e., of the variable S) is given by $\phi(z) = \pi_0 + z\varphi(z)$.

We summarize the above results in the following theorem.

Theorem 1. *The Markov chain* $\{X_m,\ m \in \mathbb{N}\}$ *is ergodic if and only if*

$$\rho(q^+ - q^-) < 1.$$

The generating function of the stationary distribution of the chain is given by

$$\phi(z) = \frac{1-\rho(q^+ - q^-)}{(1-2\rho q^+)z_1^* + 2\rho q^- + 1}\left[z_1^* + 1 + \frac{z}{sq^+(z_2^* - z)}\right]$$

where $z_1^* \in (-1, 0)$ *and* $z_2^* \in (0, +\infty)$ *are the solutions of the equation*

$$s\rho q^+ z^2 - (\bar{p} + \rho q^-)z - pq^- = 0.$$

Corollary 1. *The probability generating function of the number of customers in the waiting line (i.e., of the variable* N*) is given by*

$$\psi(z) = \pi_0 + \varphi(z) = \frac{1-\rho(q^+ - q^-)}{(1-2\rho q^+)z_1^* + 2\rho q^- + 1}\left[z_1^* + 1 + \frac{1}{sq^+(z_2^* - z)}\right].$$

After a simple derivation exercise, we obtain

$$\varphi^{(k}(z) = \frac{1-\rho(q^+ - q^-)}{(1-2\rho q^+)z_1^* + 2\rho q^- + 1}\,\frac{k!}{sq^+(z_2^* - z)^{k+1}}, \quad k \geqslant 0.$$

Corollary 2. (1) *The steady-state distribution of the waiting line size is given by the following formulae*

$$P[N=0] = \pi_0 + \pi_1 = \frac{1-\rho(q^+ - q^-)}{(1-2\rho q^+)z_1^* + 2\rho q^- + 1}\left(1 + \frac{1-\bar{s}q^-}{sq^+z_2^*}\right)$$

$$P[N=k] = \pi_{k+1} = \frac{1-\rho(q^+ - q^-)}{(1-2\rho q^+)z_1^* + 2\rho q^- + 1}\,\frac{1}{sq^+(z_2^*)^{k+1}}, \quad k \geqslant 1.$$

(2) *The steady-state distribution of the system size is given by the following formulae*

$$P[S=0] = \pi_0 = \frac{1-\rho(q^+ - q^-)}{(1-2\rho q^+)z_1^* + 2\rho q^- + 1}(z_1^* + 1)$$

$$P[S=k] = \pi_k = \frac{1-\rho(q^+ - q^-)}{(1-2\rho q^+)z_1^* + 2\rho q^- + 1}\,\frac{1}{sq^+(z_2^*)^{k}}, \quad k \geqslant 1.$$

Corollary 3. (1) *The factorial moments of* k*th order for the waiting line size are given by*

$$\begin{aligned}\alpha_k &= E[N(N-1)(N-2)\cdots(N-k+1)]\\ &= \frac{1-\rho(q^+ - q^-)}{(1-2\rho q^+)z_1^* + 2\rho q^- + 1}\,\frac{k!}{sq^+(z_2^* - 1)^{k+1}}, \quad k \geqslant 1.\end{aligned}$$

(2) *The factorial moments of kth order for the system size are given by*

$$\beta_k = E[S(S-1)(S-2)\cdots(S-k+1)]$$
$$= \frac{1-\rho(q^+-q^-)}{(1-2\rho q^+)z_1^* + 2\rho q^- + 1}\,\frac{k!}{sq^+(z_2^*-1)^{k+1}}\,z_2^*,\ k \geqslant 1.$$

Remark 1. As we expected, the steady-state distribution for queue length can exist even when the positive arrival rate is greater than the service rate.

Remark 2. If $q^+ - q^- \leqslant 0$ (i.e., $0 < q^+ \leqslant \frac{1}{2}$ or $\frac{1}{2} \leqslant q^- < 1$), the system is stable for any values of p and s.

Remark 3. Let us observe that $\beta_k = \alpha_k z_2^*,\ k \geqslant 1$.

Remark 4. As we expected, $\beta_1 = \alpha_1 + \varphi(1)$.

4. The DST killing discipline

In this section we consider a type of clearing mechanism, called disaster, which removes all workload in the system whenever it occurs to the system. That is, the arrival of a disaster not only destroys all the unfinished work but also breaks down the server. We can think of a disaster as a machine breakdown which causes all the jobs in the system to be lost. For example, if a public telephone breaks down, all the customers leave the telephone booth; besides, the clearing mechanism of disasters can be applied to computer systems in the presence of a virus as a clearing operation of all stored messages present in the system.

The one-step transition probabilities $p_{k',k} = P[X_{m+1} = k | X_m = k']$ are given by the formulae

$$p_{0,0} = \bar{p},$$
$$p_{1,0} = \bar{s}\bar{p} + spq^-,$$
$$p_{j,0} = pq^-,\ j \geqslant 2,$$
$$p_{0,1} = p,$$
$$p_{1,1} = \bar{s}p + s\bar{p},$$
$$p_{2,1} = s\bar{p},$$
$$p_{k-1,k} = spq^+,\ k \geqslant 2,$$
$$p_{k,k} = \bar{s}pq^+ + s\bar{p},\ k \geqslant 2,$$
$$p_{k+1,k} = s\bar{p},\ k \geqslant 2,$$

where $\bar{p} = 1 - p$.

The Kolmogorov equations are

$$\pi_0 = \bar{p}\pi_0 + (\bar{s}\bar{p} + spq^-)\pi_1 + pq^-\sum_{j=2}^{\infty}\pi_j, \tag{9}$$

$$\pi_1 = p\pi_0 + (\bar{s}p + s\bar{p})\pi_1 + \bar{s}\bar{p}\pi_2, \tag{10}$$

$$\pi_k = spq^+\pi_{k-1} + (\bar{s}pq^+ + s\bar{p})\pi_k + \bar{s}\bar{p}\pi_{k+1},\ k \geqslant 2 \tag{11}$$

and the normalization condition is $\sum_{k=0}^{\infty}\pi_k = 1$.

Combining the Eq. (9) with the normalization condition, we get

$$\rho(1+q^-)\pi_0 + (pq^- - \bar{p})\pi_1 = \rho q^- \tag{12}$$

where $\rho = p/\bar{s}$ is the load of the system.

Then, multiplying (11) by z^k and summing over k, we obtain

$$[z - (\bar{s}+sz)(\bar{p}+pq^+z)]\varphi(z) = [z - \bar{s}pq^+z - (\bar{s}+sz)\bar{p}]\pi_1 - \bar{s}\bar{p}z\pi_2.$$

And substituting (10) into the above equation yields

$$[(\bar{s}+sz)(\bar{p}+pq^+z) - z]\varphi(z) = -pz\pi_0 - \bar{s}(pq^-z - \bar{p})\pi_1. \tag{13}$$

Observe that the equation

$$(\bar{s}+sz)(\bar{p}+pq^+z) - z = 0$$

has two solutions z_3^* and z_4^*, which satisfy the inequalities $0 < z_3^* < 1$ and $z_4^* > 1$.

Inserting $z = z_3^*$ in (13), we have

$$\rho z_3^*\pi_0 + (pq^-z_3^* - \bar{p})\pi_1 = 0. \tag{14}$$

The system of Eqs. (12) and (14) has a unique solution given by

$$\pi_0 = \frac{q^-(\bar{p} - pq^-z_3^*)}{\bar{p}(1-z_3^*) + q^-(\bar{p} - pq^-z_3^*)},$$

$$\pi_1 = \frac{\rho q^- z_3^*}{\bar{p}(1-z_3^*) + q^-(\bar{p} - pq^-z_3^*)}.$$

After some algebraic manipulations, Eq. (13) can be written in the form

$$\varphi(z) = \frac{\bar{p}q^-}{\bar{p}(1-z_3^*) + q^-(\bar{p} - pq^-z_3^*)}\frac{1}{sq^+(z_4^* - z)}.$$

Deriving the above expression, we obtain

$$\varphi^{(k}(z) = \frac{\bar{p}q^-}{\bar{p}(1-z_3^*) + q^-(\bar{p} - pq^-z_3^*)}\frac{k!}{sq^+(z_4^* - z)^{k+1}},\ k \geqslant 0$$

and as result

$$\pi_k = \frac{\bar{p}q^-}{\bar{p}(1-z_3^*) + q^-(\bar{p} - pq^-z_3^*)}\frac{1}{sq^+(z_4^*)^k},\ k \geqslant 1.$$

Since the probabilities $\pi_k,\ k \geqslant 0$ are greater than zero, satisfy the Kolmogorov equations and the normalization condition, we can assure that $\{\pi_k,\ k \geqslant 0\}$ is the stationary distribution for our Markov chain. As a consequence, the system is ergodic for any values of the parameters.

We now note that the probability generating function of the number of customers in the system (i.e., of the variable S) is given by $\phi(z) = \pi_0 + z\varphi(z)$.

In summary, we have the following theorem.

Theorem 2. *The generating function of the stationary distribution of the Markov chain* $\{X_m, m \in \mathbb{N}\}$ *is given by*

$$\phi(z) = \frac{q^-}{\bar{p}(1 - z_3^*) + q^-(\bar{p} - pq^- z_3^*)}\left[\bar{p} - pq^- z_3^* + \frac{\bar{p}z}{sq^+(z_4^* - z)}\right],$$

where $z_3^* \in (0,1)$ *and* $z_4^* \in (1, +\infty)$ *are the solutions of the equation*

$$(\bar{s} + sz)(\bar{p} + pq^+ z) - z = 0.$$

Corollary 4. *The probability generating function of the number of customers in the waiting line* (*i.e., of the variable* N) *is given by*

$$\psi(z) = \pi_0 + \varphi(z) = \frac{q^-}{\bar{p}(1 - z_3^*) + q^-(\bar{p} - pq^- z_3^*)}\left[\bar{p} - pq^- z_3^* + \frac{\bar{p}}{sq^+(z_4^* - z)}\right].$$

Corollary 5. (1) *The steady-state distribution of the waiting line size is given by the following formulae*

$$P[N = 0] = \pi_0 + \pi_1 = \frac{q^-(\bar{p} - pq^- z_3^* + \rho z_3^*)}{\bar{p}(1 - z_3^*) + q^-(\bar{p} - pq^- z_3^*)},$$

$$P[N = k] = \pi_{k+1} = \frac{\bar{p}q^-}{\bar{p}(1 - z_3^*) + q^-(\bar{p} - pq^- z_3^*)}\,\frac{1}{sq^+(z_4^*)^{k+1}},\quad k \geqslant 1.$$

(2) *The steady-state distribution of the system size is given by the following formulae*

$$P[S = 0] = \pi_0 = \frac{q^-(\bar{p} - pq^- z_3^*)}{\bar{p}(1 - z_3^*) + q^-(\bar{p} - pq^- z_3^*)},$$

$$P[S = k] = \pi_k = \frac{\bar{p}q^-}{\bar{p}(1 - z_3^*) + q^-(\bar{p} - pq^- z_3^*)}\,\frac{1}{sq^+(z_4^*)^{k}},\quad k \geqslant 1.$$

Corollary 6. (1) *The factorial moments of* k*th order for the waiting line size are given by*

$$\begin{aligned}\alpha_k &= E[N(N-1)(N-2)\cdots(N-k+1)]\\ &= \frac{\bar{p}q^-}{\bar{p}(1 - z_3^*) + q^-(\bar{p} - pq^- z_3^*)}\,\frac{k!}{sq^+(z_4^* - 1)^{k+1}},\quad k \geqslant 1.\end{aligned}$$

(2) *The factorial moments of kth order for the system size are given by*

$$\beta_k = E[S(S-1)(S-2)\cdots(S-k+1)]$$
$$= \frac{\bar{p}q^-}{\bar{p}(1-z_3^*)+q^-(\bar{p}-pq^-z_3^*)}\,\frac{k!}{sq^+(z_4^*-1)^{k+1}}\,z_4^*,\ k \geqslant 1.$$

Remark 5. Let us observe that $\beta_k = \alpha_k z_4^*,\ k \geqslant 1$.

Remark 6. As we expected, $\beta_1 = \alpha_1 + \varphi(1)$.

5. Numerical examples

In this section, using numerical results, we show the influence of the parameters on several performance characteristics.

In the Figs. 1–3, we consider the RCH killing discipline choosing the parameters under the stability condition.

In Fig. 1(a), the stationary probabilities are plotted versus p. The curves are decreasing as function of k, according to the analytic expression for $\pi_k,\ k \geqslant 1$. We observe that π_k ($k \geqslant 1$), as function of p, starts increasing to a maximum and, as is to be expected, these maxima go to the right when k increases. The behaviour of the graphic for π_0 is clearly intuitive.

The effect of the parameter q^+ on the stationary probabilities is shown in Fig. 1(b). We see that, as q^+ approaches 0, the system becomes dichotomic, that is, there are only two states: either the system is empty or the system contains a unique customer.

The expected values $E[N]$ and $E[S]$, shown in the Fig. 2 as a function of the parameter p, increase with increasing values of p. Besides, the curves increase according to higher probabilities of s. When $p \approx 1$ and $s \geqslant 0.5$, the system is no longer ergodic and therefore these expectations become infinite.

Substantially the same comments of Fig. 2 are valid for Fig. 3, which illustrates the behaviour of $E[N]$ and $E[S]$ as function of q^+. Nevertheless, we observe that, as q^+ tends to 0, the expected value $E[S]$ belongs to (0,1), which agrees with the comments to the Fig. 1(b).

On the other hand, in the case of the DST discipline it can be seen that Figs. 1–3 and their comments are similar and consequently they are omitted. Although we note that the values of the involved parameters are not restricted to any condition, given that the system with disasters is always stable.

Finally, in Fig. 4, we compare both killing disciplines plotting the graphics of $E[N]$ and $E[S]$ versus p. As we expected, the expectations $E[N]$ and $E[S]$ for the RCH killing discipline are greater than the corresponding ones for the DST discipline, and the difference between both of them increases with p. Moreover, there exists an interval of values of p for which $E[N]$ and $E[S]$ are just slightly affected by the killing discipline.

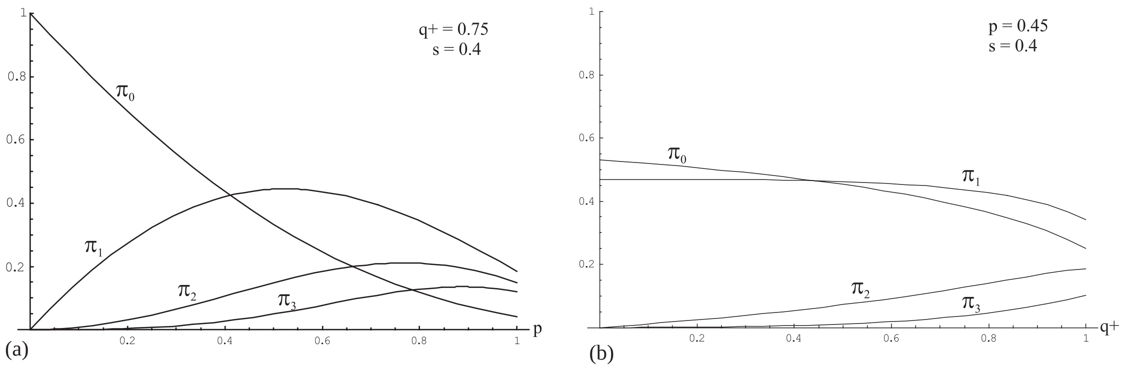

Fig. 1. The effect of p and q^+ on the stationary probabilities.

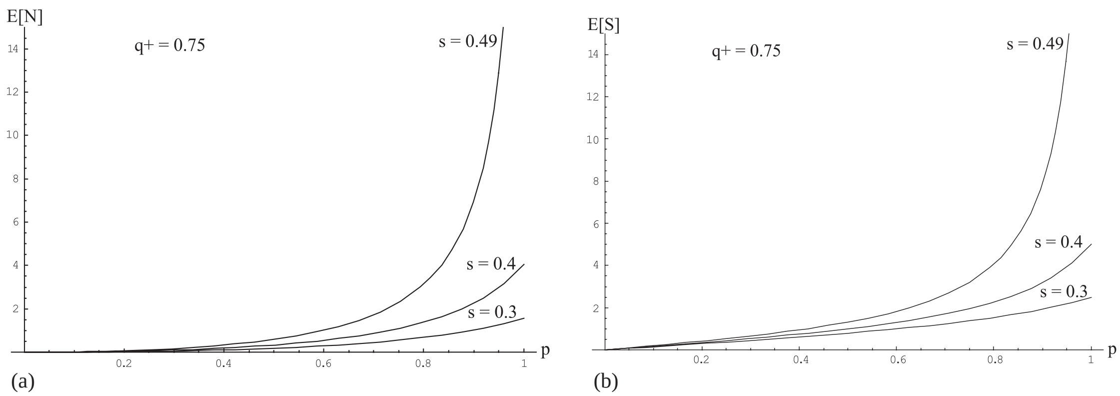

Fig. 2. The effect of p on the mean queue size and the mean system size.

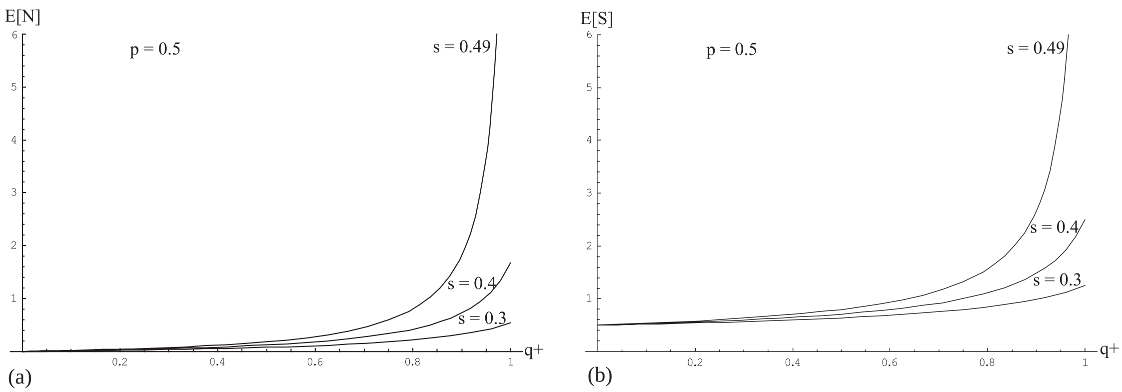

Fig. 3. The effect of q^+ on the mean queue size and the mean system size.

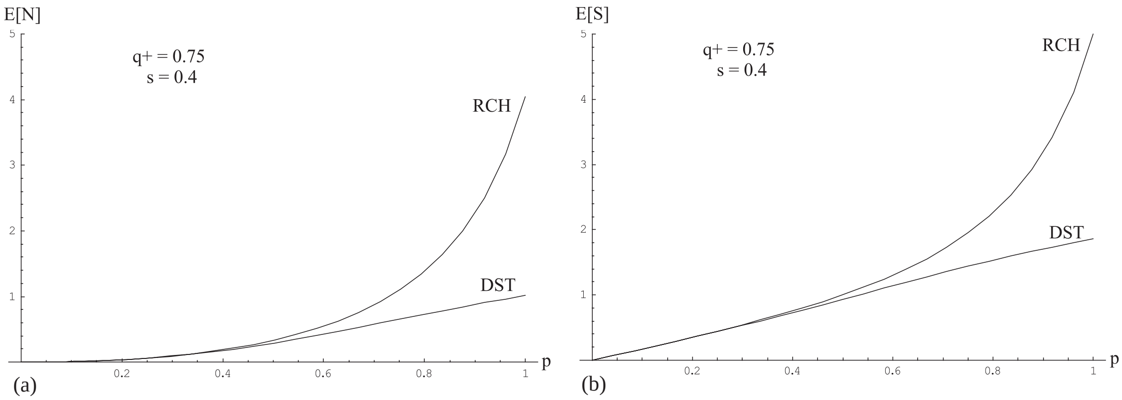

Fig. 4. The effect of p on the mean queue size and the mean system size.

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Further reading

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