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Spontaneous coherence buildup in polariton lasers

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Abstract

We derive a kinetic equation for relaxation of polaritons in a microcavity in the strong coupling regime. We show that, not making the usual Born approximation between ground and excited states, second order coherence builds up in the ground state once enough polaritons accumulate here. This comes from particles conservation, without polariton–polariton interactions.

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1. Introduction

After the Nobel prize winning observation of Bose–Einstein condensation (BEC) in cold atoms, the interest for this phase transition has spread to the condensed matter community where it is actively looked for in as many shapes and variations as could possibly be conceived. BEC in excitonic systems appear particularly promising and we take concern in one such candidate—namely microcavity polaritons [1]—with such appealing properties as, among others, a very small mass (which varies inversely to critical temperature), well defined and accessible minimum of energy (as opposed to bulk polaritons) or delocalisation (making them robust to dephasing by impurities or disorder). Not least, experimental evidence supports polaritons as efficient bosons [2]. Polaritons, however, have very short lifetime which brings the field in the out-of-equilibrium regime. Whereas the growth of condensate was previously considered from an equilibrium point of view [3,4], lifetime becomes for such short-lived particles an unavoidable issue. At best, one can resort to a steady state where lifetime is balanced by external pumping, a view we shall take here.

The dynamical description of the formation of a condensate in a bosonic system was first investigated with Boltzmann equation (BE) [5], with negative results in the sense that BE is unable to gather by itself a macroscopic amount of particles in the ground state. It nevertheless remains valid in absence of the condensate or once it has been externally introduced (as a seed) [10]. Thus, BE breaks in a so-called coherent stage [6] where the condensate ignites. In a finite size or etched system where the ground state is separated in energy from its closest excited states, this difficulty can be relaxed by linking condensation not to the stringent condition of a singularity in the distribution function, but merely to a population number greater than one in a single state. Motivation for this criterion comes from bosonic stimulation then becoming dominant over the spontaneous part.

In this paper, we derive a kinetic equation which extends to open systems considerations developed for condensation of stable bosons [3,4]. In the framework of this formalism, we are able to report spontaneous (without seed) coherence build-up in a finite size microcavity [7], as observed experimentally [8].

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2. Derivation of kinetic equation

We aim to write a kinetic equation describing the dynamics of polaritons which embeds both pumping and finite lifetime together with relaxation along the dispersion relation by diffusion with phonons. Since all these processes are dissipative (they all feature external fields like photons or phonons which are traced over), we seek a master equation of the Lindblad type for the density matrix ρ of polariton states in the reciprocal space:

$$\dot{\rho} = (\mathcal{L}_{\text{pol–ph}} + \mathcal{L}_{\tau} + \mathcal{L}_{\text{pump}})\rho \tag{1}$$

Here $\mathcal{L}$ are Liouville super-operators which describe respectively scattering (through phonons), lifetime and pumping. We do not consider polariton–polariton scattering which would add a unitary (non-dissipative) contribution $\mathcal{L}_{\text{pol–pol}} = -\mathrm{i}/\hbar[H_{\text{pol–pol}}, \rho]$ (see [9] for this term). In the following we undertake the derivation of $\mathcal{L}_{\text{pol–ph}}$ from the microscopic hamiltonian $H_{\text{pol–ph}}$ for polariton–phonon scattering. Similar procedure can be carried for $\mathcal{L}_{\tau}$ and $\mathcal{L}_{\text{pump}}$ to yield:

$$\mathcal{L}_{\tau}\rho = -\sum_{\mathbf{k}} \frac{1}{2\tau_{\mathbf{k}}}(a_{\mathbf{k}}^{\dagger}a_{\mathbf{k}}\rho + \rho a_{\mathbf{k}}^{\dagger}a_{\mathbf{k}} - 2a_{\mathbf{k}}\rho a_{\mathbf{k}}^{\dagger}) \tag{2a}$$

$$\mathcal{L}_{\text{pump}}\rho = -\sum_{\mathbf{k}} \frac{P_{\mathbf{k}}}{2}(a_{\mathbf{k}}a_{\mathbf{k}}^{\dagger}\rho + \rho a_{\mathbf{k}}a_{\mathbf{k}}^{\dagger} - 2a_{\mathbf{k}}^{\dagger}\rho a_{\mathbf{k}}) \tag{2b}$$

with $\tau_{\mathbf{k}}$ the lifetime, $P_{\mathbf{k}}$ the pump intensity in the state with momentum $\mathbf{k}$ and $a_{\mathbf{k}}$ the Bose annihilation operator for a polariton in this state. For instance, expression for lifetime comes from the quasi-mode coupling of polaritons with the photon field outside the cavity in the vacuum state (thereby linking spontaneous emission with the perturbation from vacuum fluctuations). We later neglect finite lifetime elsewhere than in ground state, where it is typically several orders of magnitude shorter because of dominant photon fraction. In our simulations, pumping injects excitons 10 meV above the bottom of the bare exciton band which we model by non-zero value of $P_{\mathbf{k}}$ for a collection of $\mathbf{k}$-states normally distributed about a high momentum mean value. Expression (2b) describes an incoherent pumping provided by a reservoir which pours particles in the system but does not allow their coming back. Its effect is thus merely to populate the system with incoherent polaritons, which will relax towards the ground state where they might join in a coherent phase before escaping the cavity by spontaneous emission (the light thus emitted retaining this coherence).

We pay special attention to $\mathcal{L}_{\text{pol–ph}}$ which contains the key-ingredients of our results. In interaction picture, polariton–phonon diffusion Hamiltonian reads:

$$H_{\text{pol–ph}} = \sum_{\mathbf{k},\mathbf{q}\neq\mathbf{0}} \mathcal{V}_{\mathbf{q}} e^{\frac{\mathrm{i}}{\hbar}(E_{\text{pol}}(\mathbf{k}+\mathbf{q})-E_{\text{pol}}(\mathbf{k})-\hbar\omega_{\mathbf{q}})t} a_{\mathbf{k}+\mathbf{q}}a_{\mathbf{k}}^{\dagger}b_{\mathbf{q}}^{\dagger} + \text{h.c.} \tag{3}$$

with $\mathcal{V}_{\mathbf{q}}$ the interaction strength, E_{pol} the lower polariton-branch dispersion, $\hbar\omega_{\mathbf{q}}$ the phonons dispersion and $a_{\mathbf{q}}$, respectively, $b_{\mathbf{q}}$, the Bose annihilation operator for a polariton, respectively, a phonon, in state $\mathbf{q}$. We compute $\mathcal{L}_{\text{pol–ph}}$ starting with Liouville–Von Neumann equation for polariton–phonon diffusion, $\dot{\varrho} = -\mathrm{i}/\hbar[H_{\text{pol–ph}}, \varrho]$, where ϱ is the density matrix for polaritons and phonons. Its useful part, namely the polariton density matrix, is obtained by tracing over phonons, $\rho \equiv \text{Tr}_{\text{ph}}\varrho$. Density matrix for phonons is granted as a reservoir in equilibrium with no phase coherence nor correlations with ρ. To dispense from this reservoir we write the equation for ϱ to order two in the commutator and trace over phonons, $\dot{\rho}(t) = -1/\hbar^2 \int_{-\infty}^{t} \text{Tr}_{\text{ph}}[H_{\text{pol–ph}}(t), [H_{\text{pol–ph}}(\tau), \varrho]]\mathrm{d}\tau$. We define $\mathcal{E}_{\mathbf{k},\mathbf{q}}(t) \equiv \mathcal{V}_{\mathbf{q}} e^{\frac{\mathrm{i}}{\hbar}(E_{\text{pol}}(\mathbf{k}+\mathbf{q})-E_{\text{pol}}(\mathbf{k})-\hbar\omega_{\mathbf{q}})t}$ and for convenience we write $\mathcal{H}(t) \equiv \sum_{\mathbf{k},\mathbf{q}} \mathcal{E}_{\mathbf{k},\mathbf{q}}(t) a_{\mathbf{k}+\mathbf{q}}a_{\mathbf{k}}^{\dagger}b_{\mathbf{q}}^{\dagger}$ so that $H_{\text{pol–ph}} = \mathcal{H} + \mathcal{H}^{\dagger}$. Note that operators are time independent. Because the phonons density matrix is diagonal, $[H_{\text{pol–ph}}(t), [H_{\text{pol–ph}}(\tau), \varrho]]$ reduces to $[\mathcal{H}(t), [\mathcal{H}^{\dagger}(\tau), \varrho(\tau)]] + \text{h.c.}$, which halves the algebra. Also the conjugate Hermitian follows straightforwardly, so we are left only with explicit computation of two terms, of which the first reads:

$$[\mathcal{H}(t), \mathcal{H}^{\dagger}(\tau)\varrho] = \sum_{\mathbf{k},\mathbf{q}\neq\mathbf{0}} \sum_{1,\mathbf{r}\neq\mathbf{0}} [\mathcal{E}_{\mathbf{k},\mathbf{q}}(t) a_{\mathbf{k}+\mathbf{q}}a_{\mathbf{q}}^{\dagger}b_{\mathbf{q}}^{\dagger}, \mathcal{E}_{1,\mathbf{r}}^{*}(\tau) a_1 a_{1+\mathbf{r}} b_{\mathbf{r}}\varrho(\tau)] \tag{4}$$

which, taking the trace over phonons and calling $\theta_{\mathbf{q}}\rho \equiv \text{Tr}_{\text{ph}}\left(\varrho b_{\mathbf{q}}^{\dagger}b_{\mathbf{q}}\right)$, becomes:

$$\begin{aligned}\text{Tr}_{\text{ph}}[\mathcal{H}(t), \mathcal{H}^{\dagger}(\tau)\varrho(\tau)] &= \sum_{\mathbf{k},1,\mathbf{q}\neq\mathbf{0}} \mathcal{E}_{\mathbf{k},\mathbf{q}}(t)\mathcal{E}_{1,\mathbf{q}}^{*}(\tau)\theta_{\mathbf{q}}(a_{\mathbf{k}+\mathbf{q}}a_{\mathbf{k}}^{\dagger}a_1 a_{1+\mathbf{q}}^{\dagger}\rho(\tau) \\ &\quad - a_1 a_{1+\mathbf{q}}^{\dagger}\rho(\tau)a_{\mathbf{k}+\mathbf{q}}a_{\mathbf{k}}^{\dagger})\end{aligned} \tag{5}$$

Solving numerically this equation is a considerable task, though it has already been done recoursing to Monte-Carlo simulations [3]. Instead, we here make further approximations to reduce its simulation to a level of complexity of the same order as for Boltzmann equations: we take into account correlations between ground state and excited states only, neglecting all correlations between excited states. This is motivated by the fast particle redistribution between excited states and their rapid loss of phase correlations. Physically this means that if one particle reaches the ground state, its absence is felt to some extent in the collection of excited states in a way which ensures particle number conservation. On the opposite, redistribution of particles between excited states will be seen to obey usual Boltzmann equations which pertain to averages only. Formally, we thus neglect terms like $\langle a_{\mathbf{k}_1}a_{\mathbf{k}_2}^{\dagger}a_{\mathbf{k}_3}a_{\mathbf{k}_4}^{\dagger}\rangle$ if $\mathbf{k}_i$ involve nondiagonal elements in the excited state. For nonvanishing terms, we further allow $\langle a_{\mathbf{k}_1}a_{\mathbf{k}_2}^{\dagger}a_{\mathbf{k}_2}a_{\mathbf{k}_2}^{\dagger}\rangle = \langle a_{\mathbf{k}_1}a_{\mathbf{k}_2}^{\dagger}\rangle\langle a_{\mathbf{k}_2}a_{\mathbf{k}_2}^{\dagger}\rangle$ if neither $\mathbf{k}_1$ nor $\mathbf{k}_2$ equal 0, for otherwise we retain the unfactored

expression. Terms from (5) featuring the ground state are:

$$\sum_{\mathbf{k}\neq\mathbf{0}} \mathcal{E}_{\mathbf{k},-\mathbf{k}}(t)\mathcal{E}_{\mathbf{k},-\mathbf{k}}(\tau)^{*}\theta_{\mathbf{k}}(a_{\mathbf{0}}a_{\mathbf{0}}^{\dagger}a_{\mathbf{k}}^{\dagger}a_{\mathbf{k}}\rho(\tau) - a_{\mathbf{k}}a_{\mathbf{0}}^{\dagger}\rho(\tau)a_{\mathbf{0}}a_{\mathbf{k}}^{\dagger}) \tag{6a}$$

$$+\sum_{\mathbf{k}\neq\mathbf{0}} \mathcal{E}_{\mathbf{0},\mathbf{k}}(t)\mathcal{E}_{\mathbf{0},\mathbf{k}}(\tau)^{*}\theta_{\mathbf{k}}(a_{\mathbf{0}}^{\dagger}a_{\mathbf{0}}a_{\mathbf{k}}a_{\mathbf{k}}^{\dagger}\rho(\tau) - a_{\mathbf{0}}a_{\mathbf{k}}^{\dagger}\rho(\tau)a_{\mathbf{0}}^{\dagger}a_{\mathbf{k}}) \tag{6b}$$

Recall this expression (6a) is one part of the term inside the time integral which $\dot{\rho}(t)$ evolution. Since $\mathcal{E}_{\mathbf{k},\mathbf{q}}(t)\mathcal{E}_{\mathbf{k},\mathbf{q}}(\tau)^{*} = |\mathcal{V}_{\mathbf{q}}|^{2}\mathrm{e}^{-\frac{\mathrm{i}}{\hbar}(E_{\mathrm{pol}}(\mathbf{k}+\mathbf{q})-E_{\mathrm{pol}}(\mathbf{k})-\hbar\omega_{\mathbf{q}})(t-\tau)}$, the time integration would yield a delta function of energy (times $-\mathrm{i}/\hbar$) if ρ in (6a) was τ-independent. This delta would itself provide selection rules for allowed scattering processes through the sum over $\mathbf{k}$. That $\rho(\tau)$ time evolution is slow enough as compared to this exponential to mandate this (Markov) approximation can be checked through evaluation of the phonon reservoir correlation time, which, when the reservoir has a broad-band spectrum as in our case, is short enough to allow the approximation of $\rho(\tau)$ by $\rho(t)$. In this case, (6b) vanishes as a non-conserving energy term. Gathering other terms similar to (5) eventually gives (from now on we do not write ρ time dependence anymore, which is t everywhere):

$$\begin{aligned}\dot{\rho} = -\frac{1}{2}\sum_{\mathbf{k}\neq\mathbf{0}} W_{\mathbf{0}\to\mathbf{k}}(a_{\mathbf{0}}^{\dagger}a_{\mathbf{0}}a_{\mathbf{k}}a_{\mathbf{k}}^{\dagger}\rho + \rho a_{\mathbf{0}}^{\dagger}a_{\mathbf{0}}a_{\mathbf{k}}a_{\mathbf{k}}^{\dagger} \\ - 2a_{\mathbf{0}}a_{\mathbf{k}}^{\dagger}\rho a_{\mathbf{0}}^{\dagger}a_{\mathbf{k}}) - \frac{1}{2}\sum_{\mathbf{k}\neq\mathbf{0}} W_{\mathbf{k}\to\mathbf{0}}(a_{\mathbf{0}}a_{\mathbf{0}}^{\dagger}a_{\mathbf{k}}^{\dagger}a_{\mathbf{k}}\rho \\ + \rho a_{\mathbf{0}}a_{\mathbf{0}}^{\dagger}a_{\mathbf{k}}^{\dagger}a_{\mathbf{k}} - 2a_{\mathbf{0}}^{\dagger}a_{\mathbf{k}}\rho a_{\mathbf{0}}a_{\mathbf{k}}^{\dagger})\end{aligned} \tag{7}$$

where

$$W_{\mathbf{0}\to\mathbf{k}} \equiv \frac{2\pi}{\hbar}|\mathcal{V}_{\mathbf{k}}|^{2}\theta_{\mathbf{k}}\delta(E_{\mathrm{pol}}(\mathbf{k}) - E_{\mathrm{pol}}(\mathbf{o}) - \hbar\omega_{\mathbf{k}}) \tag{8a}$$

$$W_{\mathbf{k}\to\mathbf{0}} \equiv \frac{2\pi}{\hbar}|\mathcal{V}_{\mathbf{k}}|^{2}(1+\theta_{\mathbf{k}})\delta(E_{\mathrm{pol}}(\mathbf{k}) - E_{\mathrm{pol}}(\mathbf{o}) - \hbar\omega_{\mathbf{k}}) \tag{8b}$$

We call $p(\{n_{\mathbf{k}}\})$ the diagonal of the polariton density matrix, i.e. the dotting of ρ with $|n_{\mathbf{o}}, n_{\mathbf{k}_1}, \ldots, n_{\mathbf{k}_i}, \ldots\rangle$ the Fock state with $n_{\mathbf{k}_i}$ polaritons in state $\mathbf{k}_i$:

$$p(\{n_{\mathbf{k}}\}) \equiv \langle \ldots, n_{\mathbf{k}_i}, \ldots, n_{\mathbf{k}_i}, n_{\mathbf{o}}|\rho|n_{\mathbf{o}}, n_{\mathbf{k}_1}, \ldots, n_{\mathbf{k}_i}, \ldots\rangle \tag{9}$$

This is the probability that the system be found in configuration $\{n_{\mathbf{k}}\}$ and thus the equation of motion for $p(\{n_{\mathbf{k}}\})$ is a master equation. It parallels closely Boltzmann equation with which it shares the same transition rates (8), as given by Fermi's golden rule. It also features stimulated scattering, like the Quantum (or semi-classical) Boltzmann equation. For all these reasons, following Gardiner et al. [3], we term this equation for p a Quantum Boltzmann Master Equation (QBME). From (7) we get:

$$\begin{aligned}\dot{p}(\{n_{\mathbf{k}}\}) = -\sum_{\mathbf{k}}(W_{\mathbf{0}\to\mathbf{k}}n_{\mathbf{0}}(n_{\mathbf{k}}+1) \\ + W_{\mathbf{k}\to\mathbf{0}}(n_{\mathbf{0}}+1)n_{\mathbf{k}})p(\{n_{\mathbf{k}}\}) \\ + \sum_{\mathbf{k}}(W_{\mathbf{0}\to\mathbf{k}}(n_{\mathbf{0}}+1) + n_{\mathbf{k}}p(\{n_{\mathbf{0}}+1,\ldots,n_{\mathbf{k}}-1,\ldots\}) \\ + \sum_{\mathbf{k}}(W_{\mathbf{k}\to\mathbf{0}}n_{\mathbf{0}}(n_{\mathbf{k}}+1) + p(\{n_{\mathbf{0}}-1,\ldots,n_{\mathbf{k}}+1,\ldots\})\end{aligned} \tag{10}$$

We include back (2) in (7) (note that we could have done this at any moment) and, following our spirit, we do not solve it for the entire joint probability $p(\{n_{\mathbf{k}}\})$ but average over all excited states to retain the statistical character for the ground state only. Excited states will be described with a Boltzmann equation, thus with thermal statistics. Calling $p_{0}(n_{0}) \equiv \sum_{n_{\mathbf{k}_1},n_{\mathbf{k}_2},\ldots} p(\{n_{\mathbf{k}}\})$ the ground state reduced probability (the sum is over all states but the ground state) and $\langle n_{\mathbf{k}}\rangle_{n_{\mathbf{o}}}p_{\mathbf{o}}(n_{\mathbf{o}}) \equiv \sum_{n_{\mathbf{k}_1},n_{\mathbf{k}_2},\ldots} n_{\mathbf{k}}p(\{n_{\mathbf{k}}\})$, we get the ground state QBME equation:

$$\begin{aligned}\dot{p}_{\mathbf{o}}(n_{\mathbf{o}}) = (n_{\mathbf{o}}+1)(W_{\mathrm{out}}^{n_{\mathbf{o}}+1} + 1/\tau_{\mathbf{o}})p_{\mathbf{o}}(n_{\mathbf{o}}+1) \\ - (n_{\mathbf{o}}(W_{\mathrm{out}}^{n_{\mathbf{o}}} + 1/\tau_{\mathbf{o}}) + (n_{\mathbf{o}}+1)W_{\mathrm{in}}^{n_{\mathbf{o}}})p_{\mathbf{o}}(n_{\mathbf{o}}) \\ + n_{\mathbf{o}}W_{\mathrm{in}}^{n_{0}-1}p_{\mathbf{o}}(n_{\mathbf{o}}-1)\end{aligned} \tag{11}$$

with rate transitions now function of the ground state population number $n_{\mathbf{o}}$:

$$W_{\mathrm{in}}^{n_{\mathbf{o}}}(t) \equiv \sum_{\mathbf{k}} W_{\mathbf{k}\to\mathbf{0}}\langle n_{\mathbf{k}}(t)\rangle_{n_{\mathbf{o}}} \tag{12a}$$

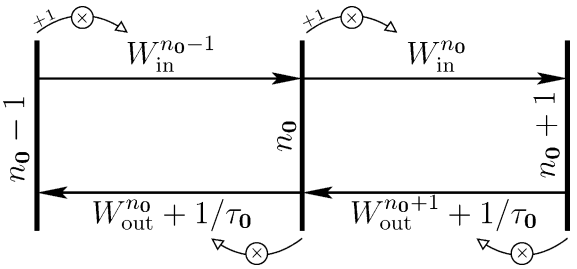
$$W_{\mathrm{out}}^{n_{\mathbf{o}}}(t) \equiv \sum_{\mathbf{k}} W_{\mathbf{0}\to\mathbf{k}}(1 + \langle n_{\mathbf{k}}(t)\rangle_{n_{\mathbf{o}}}) \tag{12b}$$

while for excited states, in Born–Markov approximation, we indeed recover Boltzmann equations:

$$\begin{aligned}\langle\dot{n}_{\mathbf{k}}\rangle = \langle n_{\mathbf{k}}\rangle\sum_{\mathbf{q}\neq\mathbf{0}} W_{\mathbf{k}\to\mathbf{q}}(\langle n_{\mathbf{q}}\rangle + 1) - (\langle n_{\mathbf{k}}\rangle + 1) \\ \times\sum_{\mathbf{q}\neq\mathbf{0}} W_{\mathbf{q}\to\mathbf{k}}\langle n_{\mathbf{q}}\rangle, \quad \mathbf{k}\neq\mathbf{0}\end{aligned} \tag{13}$$

Inclusion of $\mathcal{L}_{\tau}$ and $\mathcal{L}_{\mathrm{pump}}$ for the above adds $-\langle n_{\mathbf{k}}\rangle/\tau_{\mathbf{k}} + P_{\mathbf{k}}$ to this expression. Observe that in this case, transition rates are constants.

Cast in this form, Eq. (11) has a transparent physical meaning in terms of a rate equation for the probability of a given configuration, much like usual rate equations for occupation numbers in the framework of Boltzmann equations. The difference is that transitions from one configuration to a neighbouring one occur at rates which depend on the configuration itself, through the population of the state. This is illustrated by the following schema providing transition rates between ground state configurations differing by one particle:



Straight arrows refer to transition rates. Curved arrows refer to their weighting factors: outgoing processes are weighted by the initial population since those are rates per particle, while ingoing processes are weighted by initial population, as a result of bosonic stimulation, plus one for spontaneous diffusion. This could almost discredit the microscopic derivation since to write down the QBME (11) one might start from arrows rather than from an Hamiltonian. However, knowledge of (1) allows to get off-diagonal elements of ρ, which are needed to compute, e.g. the order parameter $\langle a_{\mathbf{o}}\rangle$. Although we shall not recourse to such analysis in this paper, this nevertheless shows there is some value beyond physical intuition carried by the schema.

$\langle n_{\mathbf{k}}\rangle_{n_{\mathbf{o}}}$ is a function of $n_{\mathbf{o}}$ that we estimate through a first order expansion about $\langle n_{\mathbf{o}}\rangle$. This implies that fluctuations of excited states are proportional (with opposite sign) to fluctuations of ground state:

$$\langle n_{\mathbf{k}}\rangle_{n_{\mathbf{o}}} \approx \langle n_{\mathbf{k}}\rangle_{\langle n_{\mathbf{o}}\rangle} + \frac{\partial\langle n_{\mathbf{k}}\rangle_{n_{\mathbf{o}}}}{\partial n_{\mathbf{o}}}|_{\langle n_{\mathbf{o}}\rangle}(n_{\mathbf{o}} - \langle n_{\mathbf{o}}\rangle) \tag{14}$$

$\langle n_{\mathbf{k}}\rangle_{\langle n_{\mathbf{o}}\rangle}$ is given by Boltzmann equation. Since the derivative does not depend on $n_{\mathbf{o}}$ (it is evaluated at $\langle n_{\mathbf{o}}\rangle$), we compute it by evaluation of both sides at a known value, for instance $n_{\mathbf{o}}=N$ with N the total particle number in the entire system, ground and excited states together. This gives $\partial\langle n_{\mathbf{k}}\rangle_{n_{\mathbf{o}}}/\partial n_{\mathbf{o}}|_{\langle n_{\mathbf{o}}\rangle} = \langle n_{\mathbf{k}}\rangle/(\langle n_{\mathbf{o}}\rangle - N)$ since $\langle n_{\mathbf{k}}\rangle_N = 0$ (no particles are left in excited states when they are all in the ground state). This is now only a matter of numerical simulations.

3. Results and conclusions

We compute numerically $p_{\mathbf{o}}(n_{\mathbf{o}})$ with (11) starting from vacuum in ground state, $p_{\mathbf{o}}(n_{\mathbf{o}}, t=0)=\delta_{n_{\mathbf{o}},\mathbf{o}}$ and computing simultaneously and self-consistently Boltzmann Eq. (13) for excited states. Here below are results for a model CdTe microcavity of 10 μm lateral size with one quantum well and a Rabi splitting of 7 meV at zero detuning. Highest polariton density obtained in the simulation, about $10^{10}\,\mathrm{cm}^{-2}$, remains below strong/weak-coupling transition density in CdTe. Phonon reservoir is at 6 K and scattering is aided by a residual gas of electrons. From the knowledge of $p_{\mathbf{o}}(n_{\mathbf{o}})$ one can obtain readily $\langle n_{\mathbf{o}}\rangle = \sum_{n_{\mathbf{o}}} n_{\mathbf{o}} p(n_{\mathbf{o}})$ and the second order coherence degree $g^2(0) = \sum_{n_{\mathbf{o}}} n_{\mathbf{o}}(n_{\mathbf{o}}-1)p(n_{\mathbf{o}})/\langle n_{\mathbf{o}}\rangle^2$. On Fig. 1 is plotted as a function of time a normalised coherence degree

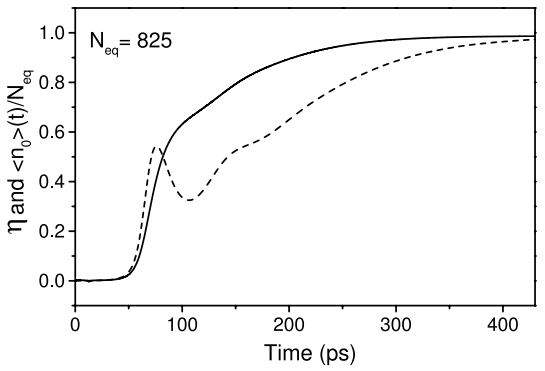


Fig. 1. Population and coherence buildup.

$\eta \equiv 2-g^2(0)$ (solid line) which better compares to $\langle n_0\rangle/N$ (dashed line) than g^2 which is the quantity measured by experimentalist with Hanbury Brown–Twiss setup. η ranges from 0 for a fully incoherent field to 1 for a coherent one. Above threshold, we observe a spontaneous (no seed, the ground state is initially empty) coherence buildup with same time order as population buildup. On Fig. 2, steady states values for coherence and (normalised) population are plotted as a function of the pump, exhibiting a dynamical threshold.

These results follow from particle number conservation: polaritons relax in energy by diffusion towards lower energy states, especially the ground state which, once it accommodates more than one polariton, triggers stimulated scattering. This is in sharp contrast with such bosons like photons or phonons which are just annihilated (being non conserved). This is the difference between a conventional laser and a polariton laser which rests on Bose–Einstein condensation. Although coherence buildup is possible thanks to any particle-conserving relaxation mechanism efficiently gathering them in a single state—as for instance the simplest channel of diffusion by phonons—a proper description of phase and order parameter requires polariton–polariton interactions. This is the topic of future publication.

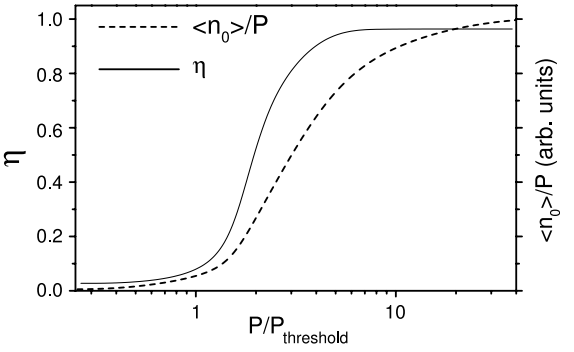


Fig. 2. Steady state population and coherence degree as a function of pumping.

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