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Negative refraction in 1D photonic crystals

Ragini Srivastava [a,*], Khem B. Thapa [b], Shyam Pati [a], S.P. Ojha [c]

[a] *Department of Physics, Udai Pratap Autonomous College, Varanasi, 221002, India*

[b] *Department of Physics, U.I. E.T, C. S. J. M. University, Kanpur-208024, India*

[c] *C. C. S. University, Meerut- 200005, India*

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ABSTRACT

Negative refraction has been the subject of considerable interest and it may provide the possibility of a variety of novel applications. It occurs in Meta-materials which have simultaneous negative permittivity ε and permeability μ, where in-homogeneities are much smaller than the wavelength of the incoming radiation. Recently, it has been shown that photonic crystals (PCs) may also exhibit negative refraction, although they have a periodically modulated positive permittivity ε and permeability μ. We have theoretically studied the negative refraction in one-dimensional (1D) photonic crystals (PCs) consisting of dielectric ZnSe with air. By using transfer matrix method and Bloch theorem, we have studied the photonic band structure and group velocity, and with the help of group velocity, we have obtained the frequency bands of negative refraction. We found that negative refraction may occur near the low frequency edge of the second and fourth bandgaps.

1. Introduction

The optical properties of materials that are transparent to electromagnetic (EM) waves can be characterized by an index of refraction which is given by the Maxwell's relation $n = \sqrt{\varepsilon.\mu}$ where ε is the relative dielectric permittivity and μ is the relative permeability of the medium. Generally, ε and μ both are positive for ordinary materials. While ε may be negative for some materials but any natural materials with negative μ are not known.

For certain structures, which are called Meta-materials, the effective permittivity and permeability, possess negative values. It means that in such materials the index of refraction is less than zero. Therefore in these materials phase and group velocity of an electromagnetic (EM) wave can propagate in opposite directions. This phenomenon is called the negative index of refraction and was theoretically proposed by Veselago [1]. Furthermore, light incident from a conventional right-handed material on Meta-materials, will bend to the same side as the incident beam and to hold Snell's law, the refraction angle should be negative.

A periodic array of artificial structures, called split ring resonators (SRRs) suggested by Pendry et al. [2], exhibits negative effective μ for frequencies close to the magnetic resonance frequency. Smith et al. [3,4] reported the experimental demonstration of meta-materials by stacking SRR and thin wire structures as arrays of 1D and 2D structured composite meta-materials (CMM). Experimental observation of negative refraction in meta-materials is verified by Shelby et al. [5]. Recently, negative refraction has been the subject of considerable interest, which may provide the possibility of a variety of novel applications of very interesting phenomena, like the super-lens effect [6–10]. A negative refractive index occurs in meta-materials that have simultaneous negative permittivity ε and permeability μ, where inhomogeneities are much smaller than the wavelength of the incoming radiation and it has been demonstrated at microwave wavelengths [11–13]. On the other hand, it has been shown that this fantastic phenomenon may also occur in ordinary materials, which are called photonic crystals (PCs) at optical wavelengths [14–17].

Photonic crystals are artificial structures, which have periodic dielectric structures with high index contrast. The resultant photonic dispersion exhibit a band nature analogous to the electronic band structure in a solid, and the propagation of electromagnetic waves are forbidden in the photonic band gap (PBG). These also show an extraordinary strong nonlinear dispersion at wavelengths close to the bandgap. Under certain conditions, they abnormally refract the light as if they had a negative refractive index, which is referred to as negative refraction effect. [18–21]. In photonic crystals, light travels as

* Corresponding address: 17/3, G-5 Maya Sadan, Krishna nagar colony, Varanasi, 221007 U.P, India. Tel.: +91 9839927479.
E-mail address: ragini_upc@yahoo.co.in (R. Srivastava).

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Bloch waves, which travel through the crystals with a definite propagation direction, despite of the presence of scattering. In the long wavelength regime, average direction of energy propagation can be shown to be the same as the direction of group velocity. [22]

The realisation of negative refraction and its experimental verification has been done by Valanju et al. [23]. They have shown that the transmission of energy is possible when the wave comes in a range of frequencies, which combine to form energy packets. A wave having just one frequency component can be bent in the wrong way, but this is irrelevant because real light never has just one frequency. Cubukcu et al. has been first to demonstrate negative refraction phenomena in two-dimensional (2D) photonic crystals in the microwave regime [24]. Further experimental studies have proved that carefully designed photonic crystals are used for obtaining negative refraction at microwave (18) and infrared [25] frequency regimes. The super-prism effect is another exciting property arising from the photonic crystals [19,26]. Extensive numerical [21,27–29] and experimental studies [30–32] have provided a better understanding of negative refraction, focusing and sub-wavelength imaging in photonic crystal structures. Masaya Notomi[33] showed that refraction-like behaviour could be expected to occur in photonic crystals that exhibit negative refraction for certain lattice parameters.

Boedecker and Henkel [34] mentioned that the simple one-dimensional Kronig–Penney model provided an exactly soluble example of a photonic crystal with negative refraction. In this paper, we have studied the negative refractive behaviours in 1D photonic crystal by the Bloch theory and transfer matrix method. With the help of the group velocity and the transmittance, we get frequency bands of negative refraction. When negative refraction occurs in photonic crystals, the transmission wave will shift adversely from the incidence point in x-axis to the end face.

2. Theoretical analysis

To study the propagation of electromagnetic waves in one-dimension photonic crystal, let us consider a 10 period structure of alternating layers of two materials with different refractive index contrast n_1 and n_2 respectively, a and b are the widths of two layers respectively and $d = a + b$ (Fig. 1).

Suppose an electromagnetic wave in the long wavelength falls obliquely on the interface of a one-dimension photonic crystal with an incident angle θ_0. We assume that the wave vector has components only in the x and z directions, then according to the Bloch wave theory, the dispersion relation for this periodic dielectric layers is given by

$$K = \frac{1}{d}\cos^{-1}\left(\cos(k_1.a).\cos(k_2.b) - \frac{1}{2}\left(\frac{k_2}{k_1} + \frac{k_1}{k_2}\right) \times \sin(k_1.a).\sin(k_2.b)\right) \quad (1)$$

where $k_i = \left(\left(\frac{\omega}{c}n_i\right)^2 - \beta^2\right)^{\frac{1}{2}} = \frac{n_i.\omega}{c}\cos(\theta_i)$, $\theta_i = \sin^{-1}\left(\frac{n_0.\sin\theta_0}{n_i}\right)$ and $i = 1, 2$

Let the transformations $t_1 = a/d$ and $t_2 = b/d$ and normalizing the frequency $\bar{\omega} = \frac{\omega.d}{c}$ we obtain the expression for the dispersion relation as

$$K = \frac{1}{d}\cos^{-1}(\cos(n_1.t_1\cos\theta_1\bar{\omega}).\cos(n_2.t_2\cos\theta_2\bar{\omega}) - \gamma.\sin(n_1.t_1\cos\theta_1\bar{\omega}).\sin(n_2.t_2\cos\theta_2\bar{\omega})) \quad (2)$$

where $\gamma = \frac{1}{2}\left(\frac{n_1\cos\theta_1}{n_2\cos\theta_2} + \frac{n_2\cos\theta_2}{n_1\cos\theta_1}\right)$.

The reflection and transmission can be related easily between the plane wave amplifications.

$$\begin{pmatrix} t \\ 0 \end{pmatrix} = M\begin{pmatrix} 1 \\ r \end{pmatrix}. \quad (3)$$

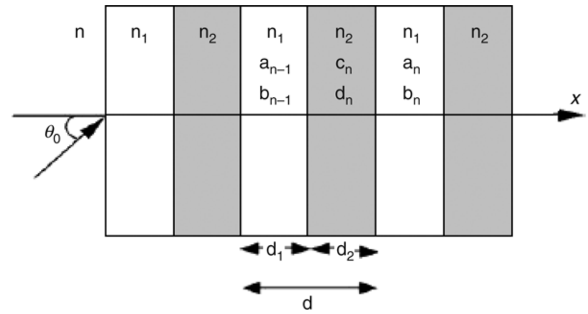

Fig. 1. Periodic refractive index profile of the structure having refractive indices n_1 and n_2 respectively.

And $M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$ with $M_{11} = m_{11}U_{N-1} - U_{N-2}$, $M_{21} = m_{21}U_{N-1}$, $M_{12} = m_{12}U_{N-1}$, $M_{22} = m_{22}U_{N-1} - U_{N-2}$ and $U_N = \frac{\sin[(N+1).K(\omega).d]}{\sin[K(\omega).d]}$ and transmittance coefficient

$$t = M_{11} - \frac{M_{12}.M_{21}}{M_{22}}. \quad (4)$$

In the photonic crystal only group velocity (V_g) has a proper meaning and it governs the energy flow of a light beam. The Bloch wave vector in photonic crystal is given by $k = \beta.\hat{x} + K.\hat{z}$. So the group velocity (V_g) in photonic crystal is expressed as [35]

$$V_g = V_{gx}.\hat{x} + V_{gz}.\hat{z},$$

where V_{gx} and V_{gz} are the components of the group velocity in the x- and z-axis respectively.

$$V_{gx} = \frac{\partial\omega}{\partial\beta} = -\frac{\partial K/\partial\beta}{\partial K/\partial\omega} \quad \text{and} \quad V_{gz} = \frac{\partial\omega}{\partial K} = \frac{1}{\partial K/\partial\omega}.$$

And resultant group velocity is

$$V_g = \sqrt{(V_{gx})^2 + (V_{gz})^2}. \quad (5)$$

The x component of the group velocity V_{gx} may cause the transverse position shift S_x along x-axis after a beam passes through photonic crystal, and is defined as [35]

$$S_x = V_{gx}.\tau \quad (6)$$

where τ is the group delay time through the structure and

$$\tau = \frac{\partial}{\partial\omega}\tan^{-1}\left(\frac{\mathrm{Im}(T)}{\mathrm{Re}(T)}\right). \quad (7)$$

3. Result and discussion

For the numerical calculation, we have taken air as dielectric with ZnSe. The refractive indices of air and ZnSe are $n_1 = 1$ & $n_2 = 2.3$ and the thicknesses are $a = 0.75\%$ of d and $b = 0.25\%$ of d respectively, where d is the total stack thickness. The total number of layers $N = 10$.

Fig. 2 shows the photonic band structure verses normalized frequency for oblique incidence $\theta = 45°$. It is clear from Fig. 2 that the bandwidth of odd number bandgap is wide but the bandwidth of even number bandgap is extremely narrow. It is noticeable here that the lattice constant d is arbitrary; thus the result obtained here is valid for arbitrary wavelengths and the existence of bandgap is possible as long as $d \cong \lambda$.

Fig. 3 shows the x-component of group velocity V_{gx} verses normalized frequency. On comparing Figs. 2 and 3, it is clear that there is a strong group velocity dispersion at the bandgap edge. With the increase of frequency, V_{gx} decreases from a positive to negative value and falls sharply to the negative minimum at the edge of band and then jumps to the positive maximum. Afterwards

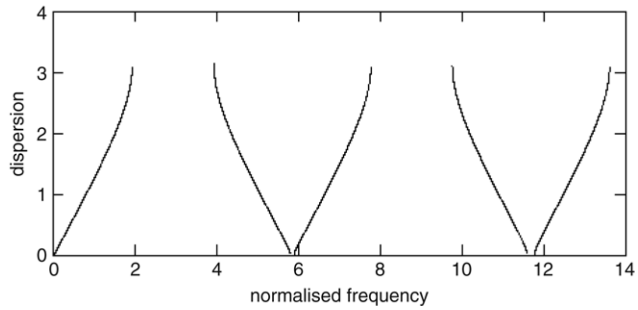

Fig. 2. Dispersion Vs normalized frequency for $a = 0.75$ and $b = 0.25$ and refractive index of air/ZnSe is $n_1 = 1$ and $n_2 = 2.3$ at angle $= 45°$.

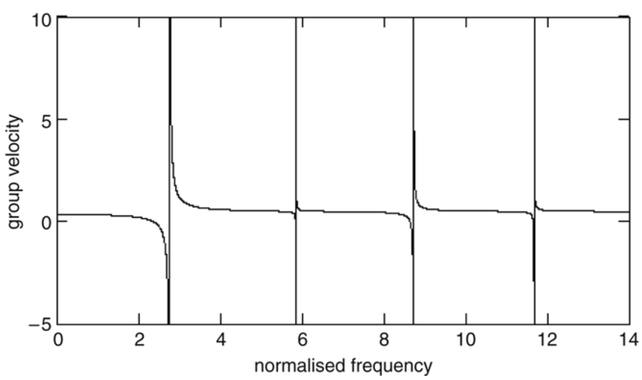

Fig. 3. Group velocity Vs normalized frequency for $a = 0.75$ and $b = 0.25$ and refractive index of air/ZnSe is $n_1 = 1$ and $n_2 = 2.3$ at angle $= 45°$.

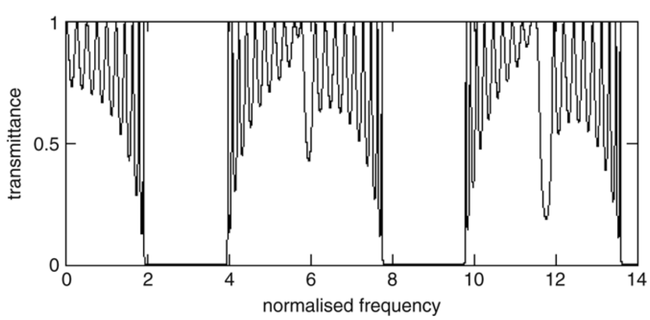

Fig. 4. Transmittance Vs normalized frequency for $a = 0.75$ and $b = 0.25$ and refractive index of air/ZnSe is $n_1 = 1$ and $n_2 = 2.3$ at angle $= 45°$.

V_{gx} decreases again and cycles in this manner. $V_{gx} < 0$ indicates that the energy flow may tend to the negative direction of x-axis, so negative refraction phenomenon may occur at some frequency in the bandgap. At normal incidence V_{gx} always equals zero, hence the oblique incidence of the wave is the necessary condition for negative refraction. From Fig. 4, we observe that the transmittance is zero in the first and third bandgap, i.e. at these frequencies the waves are completely reflected and do not pass through the crystal but in the second and fourth bandgaps, the transmittance is not zero, which means part of the wave can pass. Relating with Fig. 3, we observe that there exists a negative group velocity in the low frequency edge of the second bandgap, at the same time; the transmittance is not zero, so the transmission wave will bend to the negative direction of the x-axis, which is the negative refraction phenomenon. According to the parameters in Fig. 3, the normalized frequency band for negative refraction lies between 5.722 and 5.738 in the second band gap and 11.528 and 11.542 in the fourth band gap, but the transmission energy is less than 5%. However, in the first and third bandgaps, some group velocity is negative, so there is no transmission of energy and the negative refraction

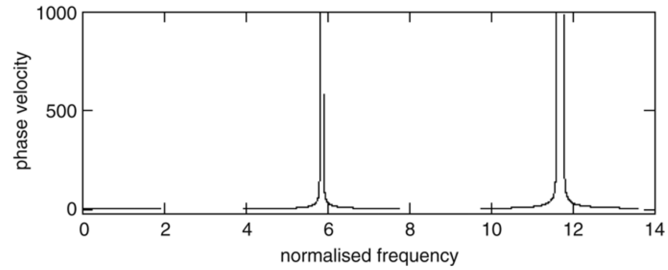

Fig. 5. Phase velocity Vs normalized frequency for $a = 0.75$ and $b = 0.25$ and refractive index of air/ZnSe is $n_1 = 1$ and $n_2 = 2.3$ at angle $= 45°$.

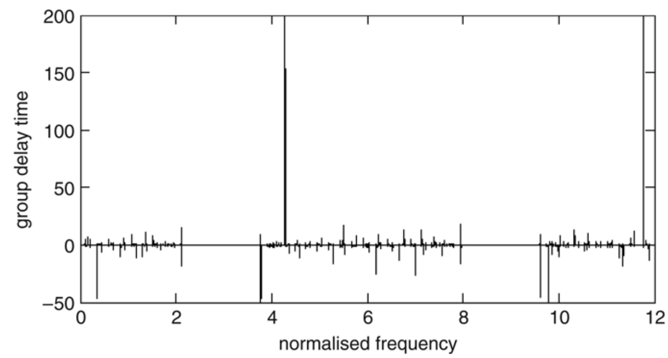

Fig. 6. Group delay time Vs normalized frequency for $a = 0.75$ and $b = 0.25$ and refractive index of air/ZnSe is $n_1 = 1$ and $n_2 = 2.3$ at angle $= 45°$.

can not occur. Therefore, we can conclude that negative refraction may occur in the frequency near the low frequency edge of the second and fourth bandgap because of the strong group velocity dispersion. Fig. 5 shows the phase velocity verses normalized frequency. The phase velocity in the second and fourth bandgaps is infinite because in this region the wave is evanescent, the wave number is complex and only the imaginary part exists. Whereas in theory, the evanescent wave decays to zero in infinite photonic crystal, so the phase of the wave does not exist therefore phase velocity has no meaning. In the first and third bandgaps, the wave number is complex, but the real part is not zero, so the phase velocity exists, and the phase of the wave varies because of the reflection at interface boundary, the waves are all reflected and can not pass through photonic crystal. From Fig. 6, it is clear that the group delay time in the first and third bandgap is zero, but in the second and fourth bandgaps it is obvious. Therefore, we can conclude that negative refraction may occur in the frequency near the low frequency edge of the second and fourth bandgap because of the strong group velocity dispersion.

4. Conclusions

In conclusion, we demonstrated theoretically that negative refraction may occur near the low frequency edge of the second and fourth bandgaps in 1D photonic crystals for an oblique incidence of wave. These unique properties of refracting Bloch photons have the potential to perfect the design of integrated photonic systems.

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