Superlattices and Microstructures, Vol. 22, No. 2, 1997

Magneto-transport of electrons in a nonhomogeneous magnetic field

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(Received 15 July 1996)

We investigate the magnetoresistance of a two-dimensional electron gas (2DEG) in the presence of a magnetic tunnel barrier of μm width. The classical 2D electrostatic problem is solved numerically from which we obtain the potential, electric field and current distribution in the 2DEG. We found that most of the electrons are injected at the edges of the magnetic barrier. Our results reproduce the positive background resistance observed by M. L. Leadbeater *et al*. [Phys. Rev. **B52**, R8629 (1995)] in the magnetoresistance of a nonplanar GaAs/AlGaAs heterojunction.

Key words: magnetotransport, two-dimensional electron gas, GaAs/$Al_xGa_{1-x}As$.

1. Introduction

The many different and creative new environments under which the two-dimensional electron gas (2DEG) is investigated have greatly improved our understanding of these systems and has led to the observation of several remarkable phenomena such as the integer and the fractional quantum Hall effects [1], conductance quantization in quantum point contacts, the Aharonov–Bohm effect, and the Weiss oscillations to mention just a few. Recently, an increasing amount of effort was devoted to the experimental and theoretical investigation of the behavior of the 2DEG under the influence of a nonhomogeneous magnetic field [2]. Such nonhomogeneous magnetic field profiles can be produced by, e.g. depositing lithographic patterned superconducting or ferromagnetic films on top of a heterojunction.

Recently, Leadbeater *et al*. [3] reported an alternative technique to produce spatially varying magnetic fields. They constructed a non-planar two-dimensional electron gas (2DEG) which was fabricated by growth of a GaAs/(AlGa)As heterojunction on a wafer pre-patterned with facets at 20° to the substrate. Applying a uniform magnetic field (B) produces a spatially non-uniform field component perpendicular to the 2DEG (see inset in Fig. 1). With the field in the plane of the substrate an effective magnetic barrier is created located at the facet. The resistance measured across such an etched facet showed oscillations which are periodic in $1/B$, and which are on top of a positive magnetoresistance background which increases quadratically with the magnetic

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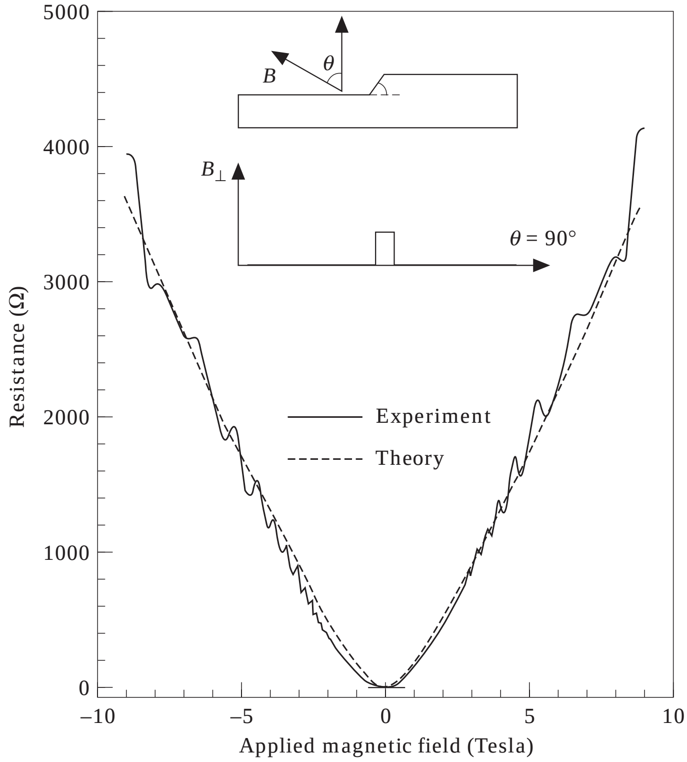

Fig. 1. The magnetoresistance as a function of the external magnetic field. The inset shows the experimental system together with the magnetic field profile perpendicular to the 2DEG.

field for small B and linear in B for large B. For the experimental trace shown in Fig. 1 the dimensions of the facet are 40 μm wide and 3 μm long, the voltage probes are situated 10 μm apart across the facet (see Ref. [3] for more details).

2. Theoretical model

To explain, quantitatively, the main features of these experimental measurements, namely the smooth background of the magnetic field dependence of the resistance, we will rely on a classical model. In order to calculate the spatial distribution of the electrostatic potential, the electric field and the current density, we start with the following set of equations

$$\nabla \times \boldsymbol{E}(x, y) = 0, \tag{1}$$

$$\nabla \cdot \boldsymbol{J}(x, y) = 0, \tag{2}$$

which are supplemented with Ohm's law and the expression for the Lorentz force

$$\boldsymbol{J} = \sigma \boldsymbol{E}, \tag{3}$$

$$\boldsymbol{F} = e(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}), \tag{4}$$

where $\boldsymbol{F} = m(d/dt + 1/\tau)\boldsymbol{v}$, τ is the relaxation time related to the measured mobility of the sample and the term $m\,d\boldsymbol{v}/dt$ is zero in the steady state. The above equations reduce to the following elliptic equation for the electric potential ($\boldsymbol{E} = -\nabla\phi$)

$$\nabla[\sigma(x, y)\nabla\phi(x, y)] = 0, \tag{5}$$

where $\sigma(x, y)$ is a spatial dependent conductivity tensor. For $\sigma(x, y) = constant$ this equation reduces to the well known Laplace equation. In our case the conductivity tensor is no longer constant due to the presence of the magnetic barrier: $\sigma(x, y) = \alpha[\sigma_{ij}]$ where $\alpha = \sigma_0/[1+(\mu_e B)^2]$ and the components σ_{ij} are $\sigma_{xx} = \sigma_{yy} = 1$ and $\sigma_{xy} = -\sigma_{yx} = \mu_e B$ where μ_e is the mobility and $B = 0$ outside the facet. The 2D differential equation is solved numerically using a finite-difference technique with the boundary conditions $\phi(x, 0) = 0$ and $\phi(x, 0.25) = 1$, and the condition that no current can flow out the sides of the sample. The distances are normalized by the width of the sample ($W = 40\ \mu$m taken along the x-axis) and voltages are normalized by the total voltage drop between the voltage probes. The length of the sample is 10 μm and consequently $y = 0.25$ is the length of our sample in these units. The magnetoresistance is given by $R = V/I$, where we have set the total voltage drop $V = 1$, and the total current flowing normal to the facet is obtained through $I = \int_a^b j_y(x)\,dx$, where a and b are any two points on the opposite sides of the sample. In our numerical analysis we used $\sigma_0 = 1/(53.146\ \Omega)$ which is obtained from the experimentally measured resistance in Ref. [3].

3. Results and discussion

In Fig. 1 we show both the experimental (solid) and the theoretical (dashed) traces for the magnetoresistance for the case of a single facet. It is seen that apart from the Shubnikov–de Haas (SdH) oscillations, which result from the quantizing effect of the magnetic field at low temperature, the theoretical curve accounts nicely for the overall behavior of the magnetoresistance. The experimental curve is slightly asymmetric around $B = 0$ which is probably due to a slight misalignment of the voltage probes. The classical origin of the positive magnetoresistance was confirmed experimentally where it was found that it persists at temperatures higher than 100 K. Note that the experimental configuration is effectively a two terminal measurement where the measured resistance is determined both by the Hall resistance as well as the magnetoresistance. For small B-fields the Hall resistance is small and thus the resistance is determined by the magnetoresistance and consequently quadratic in B. For larger magnetic fields a quasi-linear behavior of the resistance as a function of B is found which is due to the fact that now it is the Hall resistance which mainly limits the current.

The theoretical electric potential distribution in the sample is shown in Fig. 2 for an applied magnetic field of 2 Tesla which results into $B_\perp = 0.6840$ Tesla. Notice that almost all of the potential drop takes place in the magnetic barrier. In the barrier region and just outside of it there is a voltage difference between the edges of the sample (see Fig. 3), which is nothing else than a spatially dependent Hall voltage. This is in accord with the concept that the planar regions ($B = 0$ regions) can be thought of as extended high mobility contacts to a short and wide Hall bar (the facet region) which tend to shorten out most of the voltage immediately outside the facet region. Particularly interesting is the development of the Hall voltage between the opposite edges of the facet which becomes small but nonzero outside the facet region and the steep increase of the Hall potential profile at the edges of the $B \neq 0$ region which is reminiscent of the potential profile investigated experimentally and theoretically in Refs [4, 5] in a conventional Hall bar under the conditions of the quantum Hall regime and in the middle of a plateau in the Hall resistance.

The spatial distribution of each component of the electric field can be obtained from $E_x = -\partial\phi(x, y)/\partial x$ and $E_y = -\partial\phi(x, y)/\partial y$. Thus, one can see that both components are very small outside the facet region, inside the facet E_x becomes very large close to the edges, especially at the diagonally opposite corners and vanishes in the middle where E_y is constant, singular at the diagonally opposite corners and very small at the other two corners. Accordingly, the largest part of the current will enter the facet region from the corner where both of the electric field components are large and exit the facet from the diagonally opposite corner. This type of behavior was proposed in Ref. [3] on physical grounds without any detailed calculation. Once

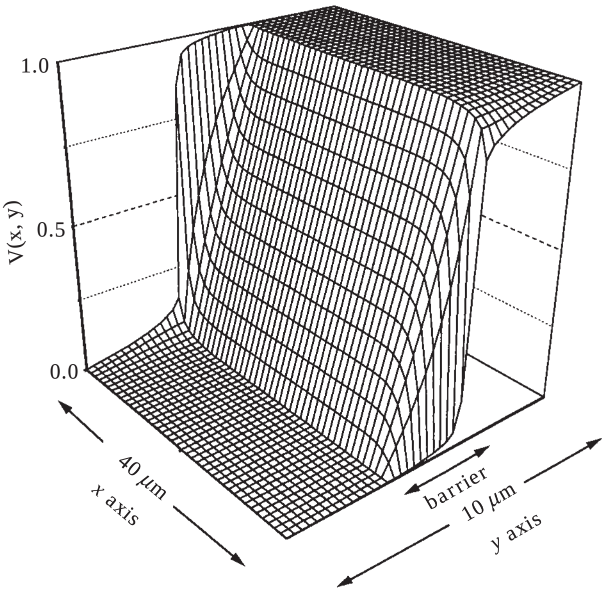

Fig. 2. The potential distribution (normalized to the applied voltage) in the sample for $B = 2$ Tesla.

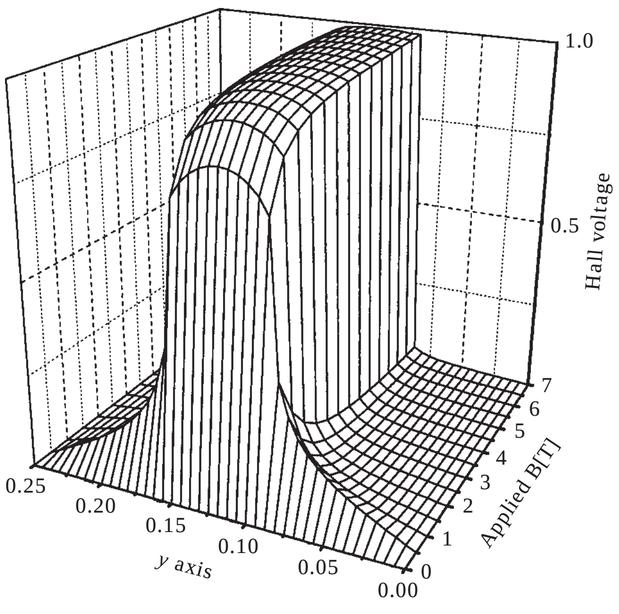

Fig. 3. The Hall voltage (normalized to the applied voltage) measured across the sample as a function of the longitudinal distance along the sample for different values of the external magnetic field.

inside the facet region the guiding center of the electron cyclotron orbits will drift along the equipotential lines (see Fig. 2) according to the $\boldsymbol{E} \times \boldsymbol{B}$ drift with velocity $\boldsymbol{v}_{drift} = -(\nabla\phi \times \boldsymbol{B})/eB^2$. Electrons entering or exiting the small regions of the facet corners will have large velocities proportional to the electric field at these location to account for current conservation with a larger number of electrons drifting with slow and

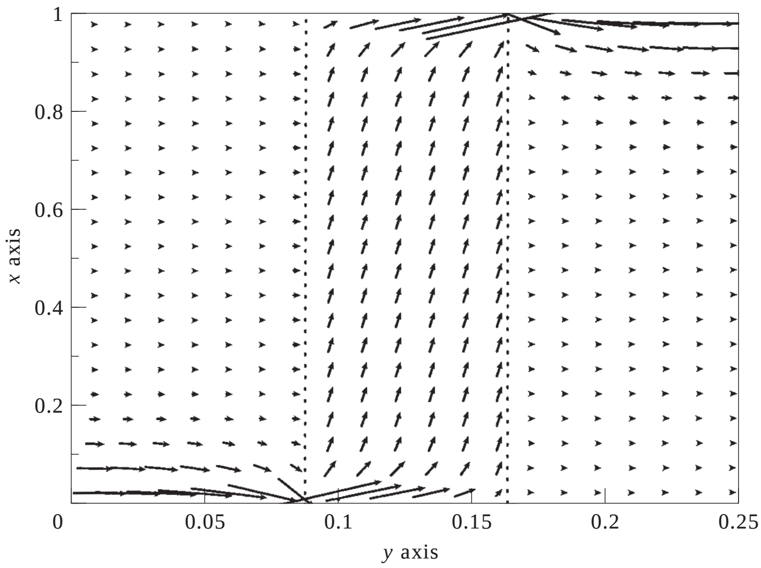

Fig. 4. The current flow in the sample for $B = 2$ Tesla. The position of the magnetic field barrier is indicated by the dotted line.

uniform velocities in the middle of the facet where the electric field is smaller and uniform. This picture is graphically represented in Fig. 4 in which we show the calculated results for the current distribution. Notice that even well outside the facet the current is already modified by the presence of the magnetic field in the facet region and it is concentrated closer to the edges of the sample. At the diagonally opposite corners it is strongly peaked. These results for the field and current distribution are consistent with those of Ref. [6] which were calculated for a conventional Hall geometry in the case of very low aspect ratio.

Acknowledgements—Part of this work is supported by the Belgian National Science Foundation, the programme INTAS-93-1495 and the Russian Foundation for Basic Research 95-02-04704.

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