Available online at www.sciencedirect.com
ScienceDirect
Physica C 468 (2008) 689–694
PHYSICA C
www.elsevier.com/locate/physc

Quantum synchronization effects in intrinsic Josephson junctions

M. Machida [a,b,*], T. Kano [a], S. Yamada [a,b], M. Okumura [a,b], T. Imamura [b,c], T. Koyama [b,d]

[a] CCSE, Japan Atomic Energy Agency, 6-9-3 Higashi-Ueno, Tokyo 110-0015, Japan
[b] CREST(JST), 4-1-8 Honcho, Kawaguchi, Saitama 332-0012, Japan
[c] University of the Electro-Communication, 1-5-1 Chofugaoka, Chofu-shiTokyo 182-8585, Japan
[d] IMR, Tohoku University, 2-1-1 Katahira Aoba-ku, Sendai 980-8577, Japan

Accepted 30 November 2007
Available online 8 March 2008

Abstract

We investigate quantum dynamics of the superconducting phase in intrinsic Josephson junctions of layered high-T_c superconductors motivated by a recent experimental observation for the switching rate enhancement in the low temperature quantum regime. We pay attention to only the capacitive coupling between neighboring junctions and perform large-scale simulations for the Schrödinger equation derived from the Hamiltonian considering the capacitive coupling alone. The simulation focuses on an issue whether the switching of a junction induces those of the other junctions or not. The results reveal that the superconducting phase dynamics show synchronous behavior with increasing the quantum character, e.g., decreasing the junction plane area and effectively the temperature. This is qualitatively consistent with the experimental result.

PACS: 74.81.Fa; 75.45.+j; 05.45Xt

Keywords: Quantum synchronization effects; Intrinsic Josephson junctions

1. Introduction

Since the discovery of "intrinsic Josephson effects" in layered high-T_c superconductors, the Josephson effects have attracted much interests from standpoints of not only fundamental interest but also device application possibility. The reason is that high-T_c superconductor crystal itself is a natural coupled array of a number of atomic-scale Josephson junctions as schematically shown in the upper panel of Fig. 1. The system has been called "intrinsic Josephson junction", in which some possibilities of synchronous electromagnetic excitations over a huge number of stacked junctions have been intensively investigated [1] because such dynamics may open a promising way toward powerful high-frequency radiation source. Moreover, the reproducibility of identical Josephson junction has been so far considered to be a crucial breakthrough point in the Josephson device production, while intrinsic Josephson junctions are completely free from such a problem if one can make good crystallines.

A historical chain of theoretical studies on the intrinsic Josephson effects have clarified an importance of the coupling between stacked neighboring junctions. The dynamics of the superconducting phase in intrinsic Josephson junctions is now well-known to be never understood without the coupling between junctions. Since the recent observation of macroscopic quantum tunneling (MQT) in intrinsic Josephson junctions [2–4], a great interest has been aroused on MQT in multi-junction stacked systems [5–7]. MQT is also expected to be strongly influenced by the coupling. However, the coupling effects on MQT have been little studied except for a few recent works [5–7]. In this paper, we focus on the quantum dynamics of intrinsic Josephson junctions. A coupling is fully incorporated and effects of the coupling on the quantum dynamics are clarified through large-scale numerical simulations.

* Corresponding author. Address: CCSE, Japan Atomic Energy Agency, 6-9-3 Higashi-Ueno, Tokyo 110-0015, Japan.
E-mail address: machida.masahiko@jaea.go.jp (M. Machida).

0921-4534/$ - see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.physc.2007.11.081

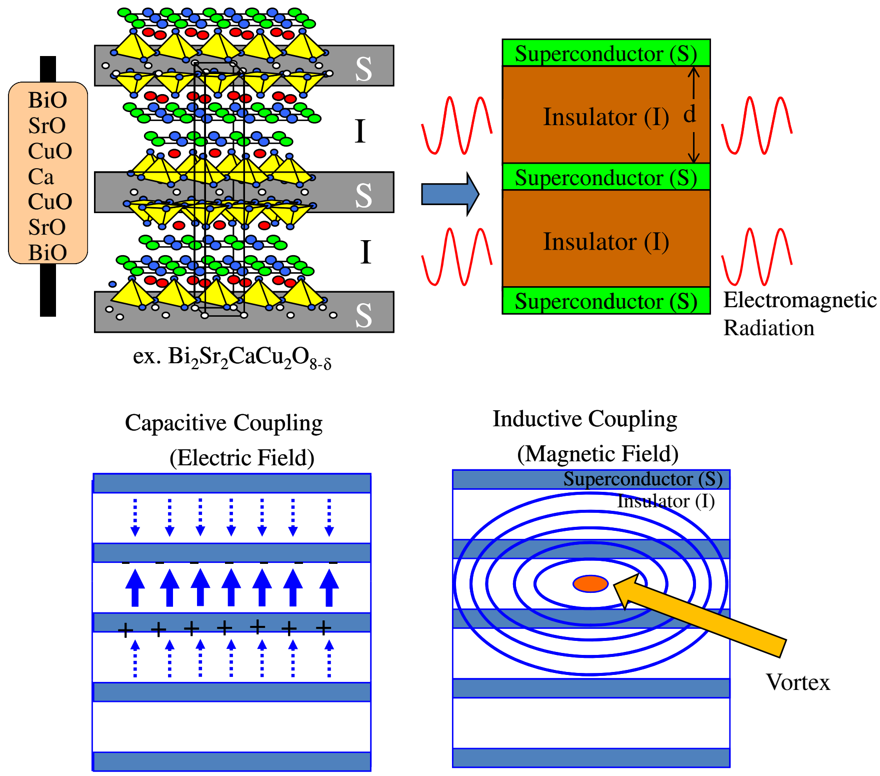

Fig. 1. A schematic figure (the upper panel) for a single crystal of layered high-T_c cuprate superconductor, which works as an intrinsic Josephson junction, in which the possibility of the in-phase electromagnetic wave radiation has been examined. There are two established couplings (the lower panel), i.e., the capacitive and the inductive coupling steming from incomplete screening for the electric and magnetic field, respectively, in intrinsic Josephson junctions.

The classical level studies classified the coupling in intrinsic Josephson junctions into two types, one of which is the capacitive coupling [8,9], and another of which is the inductive coupling. The atomic size scale of each junction in intrinsic Josephson junctions is responsible for such couplings, and the capacitive and the inductive coupling are, respectively, caused by the incomplete screening of the electric and the magnetic fields [10] as shown in the lower panel of Fig. 1. Here, we note that the former coupling is effective on relatively small junction in the zero field, while the latter one is dominant on large junction and non-zero field, i.e., the coupling becomes dominant by the penetration of the vortex (see the lower panel of Fig. 1). In this paper, we concentrate on the capacitive coupling and numerically solve the Schrödinger equation derived from the Hamiltonian considering the capacitive coupling alone. There are two reasons for ignorance of the inductive coupling in the present theoretical work. The first one comes from the previous experimental situations on MQT, in which the vortex penetration is regarded as a quite rare event since its process has a very high energy according to Ref. [11]. The second one is an explosion of the degree of freedom to be calulated. The wave function should be defined on the in-plane coordinate as well as the junction index. In Section 2 , we will show that the simulation is impossible even if we use the latest parallel supercomputer. In the present quantum simulation, we perform matrix diagonalization of the Hamiltonian matrix and directly trace the dynamics of the superconducting phase without any approximations. Instead, we face an enormous degree of freedom of the Schrödinger equation, and therefore, partly use a large-scale supercomputer, i.e., the Earth Simulator.

The MQT in intrinsic Josephson junctions has been observed by statistical measurements on the switching events into resistive states, i.e., so-called switching into “quasi-particle multi-branches”. Firstly, Inomata et al. picked up only the jump into the first branch [3], and found the crossover of the transition rate by lifting up and down the temperature. They prepaired a weak junction intentionally and regarded the results as MQT of the single weak junction. This indicates that they concentrate on intrinsic MQT properties of each piece of independent junctions for simplicity. Afterwards, Jin et al., reported that the observed MQT rates depend on the jump destinations in homogeneous samples [4]. They found that MQT rate is drastically enhanced in the case of the uniform switching, i.e., the collective switching of all junctions. Their finding is that the MQT rate is roughly proportional to the square of the number of junctions [4]. This result implies that MQT collectively occurs and the switching of some junctions induces the jump of other junctions. We expect that this synchronous behavior is caused by the coupling effect [5–7]. However, there still remains a question why such a collective MQT is enhanced only when the temperature is decreased below the crossover temperature from classical to quantum. In other words, this question is whether the quantum character assists collective or synchronous dynamics or not. In this paper, we therefore examine how the superconducting phase dynamics changes by tuning the quantum character dominance.

2. The model Hamiltonian and the Schrödinger equation

In order to examine characters of the dynamical behaviors in the quantum regime, we perform a quantum simulation for the superconducting phase of intrinsic Josephson junctions. For the purpose, the most straightfoward and exact way is to solve the Schrödinger equation derived from the model Hamiltonian. In this paper, we employ a numerically direct approach using the matrix diagonalization for the model Hamiltonian matrix. The result is exact within numerical error.

Now, let us set up the model, i.e., the Hamiltonian of the target system. The most general Hamiltonian should include both the capacitive and the inductive couplings. In this paper, we confine ourselves within small junctions in zero applied field, and drop the inductive coupling. The reduced Hamiltonian, which describes only the capcitive coupling [11], is given by

$$\widehat{H} = \sum_{\ell=1}^{N} \left\{ E_c \sum_{m=1}^{N} u_\ell \Lambda_{\ell m} u_m - E_J \cos\theta_\ell, \right\} \quad (1)$$

where u_ℓ is the canonical momentum conjugate to the phase difference, θ_ℓ, and $\Lambda_{ll} = (1 + 2\alpha)$, $\Lambda_{l\ l\pm1} = -\alpha$, and other $\Lambda_{lm} = 0$. The parameter α is the coupling constant for the capacitive coupling, which is an order of 0.1 for $Bi_2Sr_2Ca-Cu_2O_{8+\delta}$ [9,12]. The parameters E_c and E_J in Eq. (1) are defined as

$$E_c \equiv \frac{(2e)^2}{C}, \quad E_J \equiv \frac{\hbar}{2e} W j_c, \quad (2)$$

where C is the capacitance of each junction and W is the junction plane area (see Fig. 2). It is easy to check that

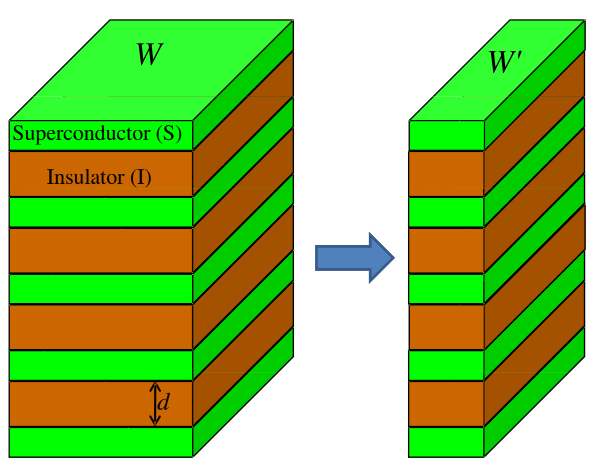

Fig. 2. With decreasing the junction plane area W, a parameter β in Eq. (3) becomes large. This is equivalent with decreasing the mass in equation of motion for a particle (see Eq. (3)). In a small range of W, the quantum behaviors are expected in the superconducting phase dynamics.

Hamilton's equation derived from the Hamiltonian (1), i.e., $\partial_t \theta_\ell(x) = \delta H / \delta u_\ell(x)$ and $\partial_t u_\ell(x) = -\delta H / \delta \theta_\ell(x)$, leads to the well-known equation of motion for the phase difference in Ref. [8,9]. The quantization can be performed by imposing the commutation relation, $[\theta_\ell(x), u_m(x')] = i\hbar \delta_{\ell m} \delta(x - x')$, for the canonical variables $(\theta_\ell(x),\ u_\ell(x))$. This relation is equivalent with the conversion $u_\ell \to -i\hbar\nabla_\ell$ in the equation of motion. Consequently, we have the following Schrödinger equation,

$$i\hbar \frac{\partial \psi}{\partial t} = \left[-\sum_{\ell} \beta(1 + 2\alpha) \frac{\partial^2}{\partial \theta_\ell^2} - \sum_{m \neq \ell} \beta\alpha \frac{\partial^2}{\partial \theta_\ell \partial \theta_m} + cos\theta_\ell \right] \psi, \quad (3)$$

where α is the coupling constant already defined in the Hamiltonian Eq. (1) and β is inversely proportional to the junction plane area W (see Fig. 2). This equation is a coupled Schrödinger equation, in which β has an role of the inverse of the mass for the Schrödinger equation describing the particle dynamics. This simply indicates that when β is small, i.e., the junction area is large, the junctions behave like classical coupled oscillators, while the junctions become quantum ones for large β (see Fig. 2). Although such a crossover from classical to quantum has been experimentally observed in intrinsic Josephson junctions [2–4] by decreasing the temperature, the changes of the collective real-time dynamics with the variation of β have not been studied theoretically. The reason is that simulating the real-time quantum evolution of the system (Eq. (3)) is quite hard. For example, as the number of coupled junctions is 4, one principally needs to solve the Schrödinger equation on 4-dimensional space if one does not use any approximations. This requires on a tremendous degree of freedom, e.g., if the phase variation of each junction is simply from 0 to 2π and the number of the grid points on the region is 256, then the total degree of freedom reaches 256^4. Then, the wave function should be solved on 256^4 grid points whose dynamics demands a massively parallel supercomputer. Thus, the real-time collective dynamics of the coupled quantum systems is a quite challenging issue for not only physics but also high-performance computing [13].

The real-time evolution of the quantum system can be described by using all eigenvalues and eigenvectors of the Hamiltonian as

$$\psi(t) = \sum_n e^{i\frac{E_n}{\hbar}t} < \psi(0) | u_n > | u_n >, \quad (4)$$

where $|u_n>$ is the eigen-function belonging to n-th eigenvalue. This expression means that if $\psi(0)$ are expanded by not all eigenfunctions but some u_n, then one does not need to obtain any u_n in order to exactly trace the time evolution. Thus, our numerical methodological issue is to select the best numerical scheme to obtain the ground state and multiple low-lying excited states in the Hamiltonian matrix diagonalization [13].

Now, let us briefly mention a scientific interest of quantum dynamics simulations for Eq. (3) or (4). The interesting issue is "Synchronization" [14]. This has been intensively investigated in a wide range of science from biology to materials science. Moreover, its research history is very long as seen in the book [14]. However, quantum effects on the phenomenon have been little investigated. In this paper, we focus on quantum effects on the phase synchronization in intrinsic Josephson junctions. It is well-known that the classical dynamics of intrinsic Josephson junctions shows independent oscillations [9], i.e., "nonlinear localized oscillations", which are in contrast to the synchronization. The consequence in the classical regime is consistent with the experiments, while the quantum regime experimentally shows the opposite behaviors. Thus, the problem can be reduced to how the quantum feature changes the localized and the independent characters. The result presented in this paper is that the quantum character assists the synchronization. We believe that this result is surprising, since the quantum character provides an opposite character contrary to the classical level.

3. Simulation results and discussion

In this section, we present simulation results and discuss their consequences. In order to perform the numerical simulations for Eq. (3) or (4), we firstly fix a variation range of the phase difference, θ_i of each junction, from 0 to 4π. Since the main purpose of the present paper is to check whether the quantum character assists synchronization or not, we prepair the following simulation. At first, an asymmetric potential (the left-hand side) as shown in Fig. 3 is set in order to initially confine the wave packet of the phases of

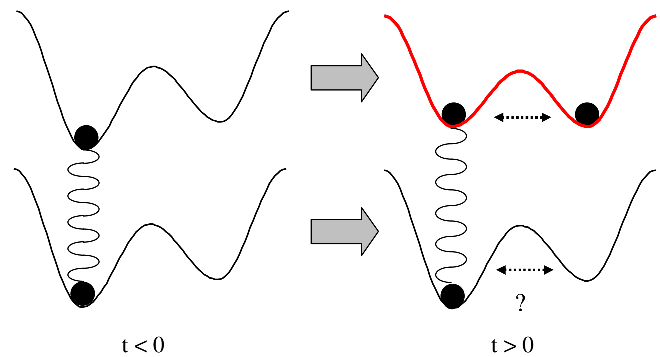

Fig. 3. A schematic figure for the potential shape change at $t = 0$. Only a potential shape is changed into the red curve (the upper panel), and the others are kept (the lower panel). (For interpretation of the references in color in this figure legend, the reader is referred to the web version of this article.)

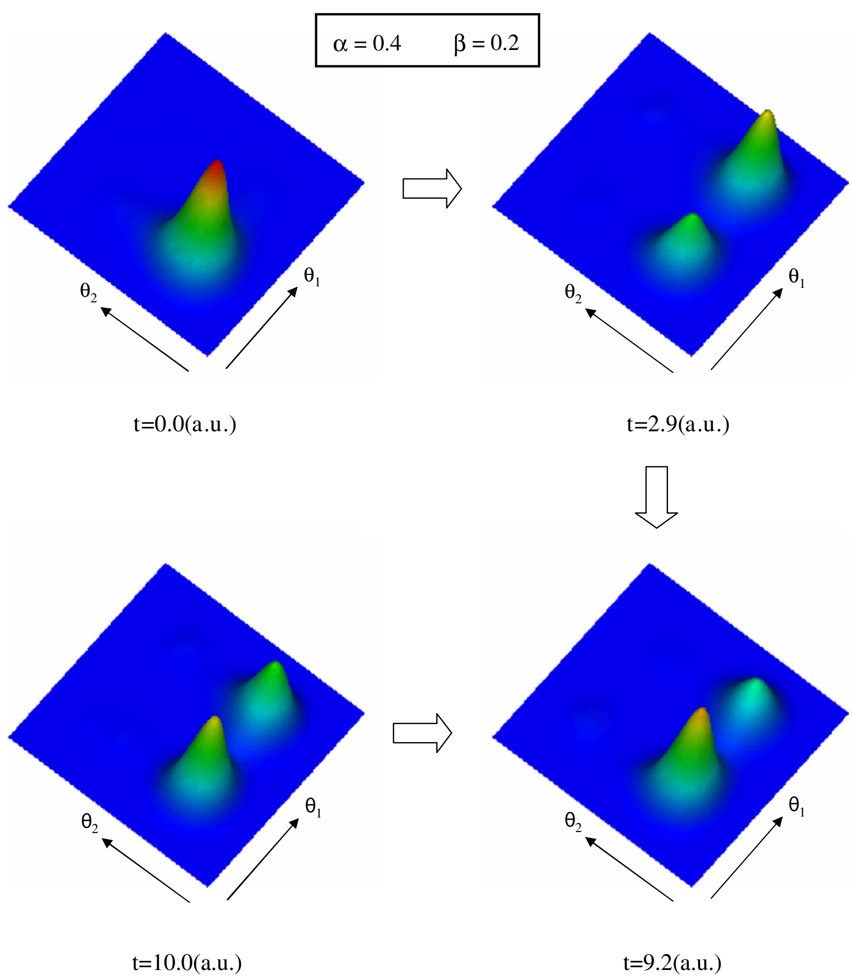

Fig. 4. A time evolution of two-coupled junction dynamics after the potential shape changes. The parameters α and β in Eq. (3) are 0.4 and 0.2, respectively.

all junctions inside a deeper potential. At $t = 0$, we change the potential for only a junction from asymmetric to symmetric shape as shown in Fig. 3. Then, the kicked junction, whose potential is changed, starts to tunnel to the neighbor potential well. The question is then whether the other junctions collectively show similar tunneling dynamics or not. If other junctions show similar dynamics, then it indicates that the tunneling of a junction induces the tunneling of others. Thus, our main issue comes down to whether the quantum feature assists the synchronous dynamics or not.

At first, let us present the simulation results in two-dimensional cases (a stacked system of two junctions). Figs. 4 and 5 shows simulation results of the time evolutions of two parameter cases, i.e., β is small (relatively classical) and large (quantum), respectively. We note that the coupling parameter $\alpha(=0.4)$ is always the same. In the former case, the wave function moves only along θ_1 axis and never shifts to θ_2 direction. From this result, it is found that only the kicked particle moves and another one keeps its position. This result indicates that only a junction can independently switch. This result is consistent with the simulations for the classical equation and the experiments in the classical range. On the other hand, Fig. 5 demonstrates a typical result of the latter quantum case. This result indicates that the kicked particle induces another particle motion. This is never observable in the simulation for the classical equation except for the large coupling. We confirm that there is a crossover range close to $\beta = 0.5$. Thus, we learn a picture that a particle motion in the quantum regime can easily induce the interacting particle's motion. We confirm that this synchronous tendency in the quantum regime also holds for the stacked systems of 3 and 4 junctions. These details are reported elsewhere [13]. Fig. 6 schematically depicts how the quantum assisted MQT in intrinsic Josephson junctions occurs. In the classical level, the switching is rather independent and the multiple branches are easily observed. This is because a classical equation of motion for the superconducting phase prefers a localized oscillating and rotating solutions [9]. On the other hand, as the junction plane area decreases, the

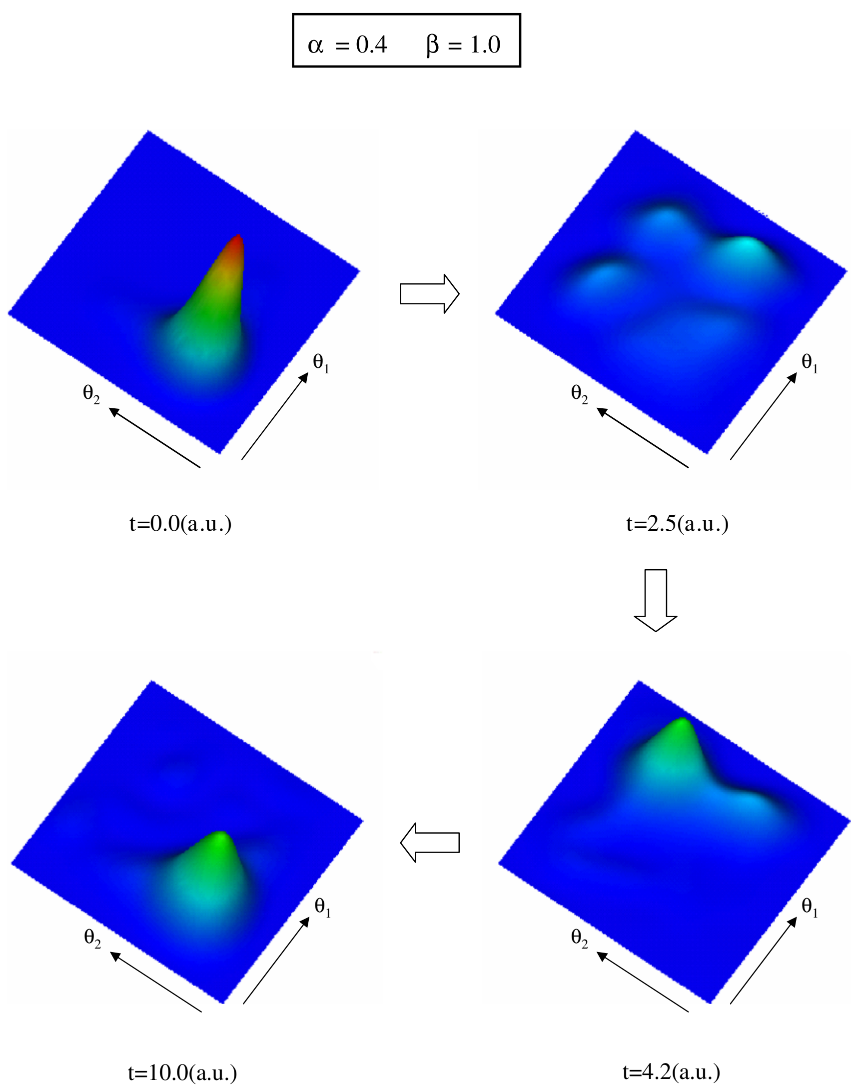

Fig. 5. The same as Fig. 4 except for the parameters $\beta(=1.0)$.

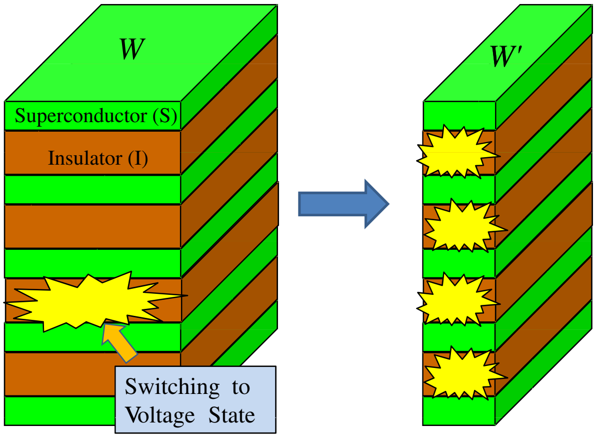

Fig. 6. A schematic figure showing a relationship between the junction plane size and the switching dynamics in intrinsic Josephson junctions.

switching becomes more collective and synchronous. The consequence is summarized in Fig. 6, where a relationship between the junction plane size and the switching dynamics is shown.

4. Summary and conclusion

We performed quantum dynamical simulations on intrinsic Josephson junctions in order to examine whether the synchronous character is enhanced in the quantum regime or not. We concentrated on the model considering only the capacitive coupling and solved the time evolution of the Schrödinger equation derived from the model Hamiltonian. The simulated situation is that only a junction is kicked, and others are left to freely evolve. The simulation results revealed that the dynamics is almost independent in the classical regime while it shows the synchronous behavior in the quantum one. These results are consistent with the experimental observations.

References

[1] See, e.g. M. Machida, T. Koyama, M. Tachiki, Physica C330 (2000) 85.
[2] T. Bauch, F. Lombardi, F. Tafuri, A. Barone, G. Rotoli, P. Delsing, T. Claeson, Phys. Rev. Lett. 94 (2005) 087003.
[3] K. Inomata, S. Sato, K. Nakajima, Y. Takano, H.B. Wang, M. Nagao, H. Hatano, S. Kawabata, Phys. Rev. Lett. 95 (2005) 107005.
[4] X.Y. Jin, J. Lisenfeld, Y. Koval, A. Lukashenko, A.V. Ustiniv, P. Muller, Phys. Rev. Lett. 96 (2006) 177003.
[5] M. Machida, T. Koyama, Supercond. Sci. Technol. 20 (2007) S23.
[6] M.V. Fistul, Phys. Rev. B 75 (2007) 014502.
[7] Sergey Savel'ev, A.L. Rakhmanov, Franco Nori, Phys. Rev. Lett. 98 (2007) 077002.
[8] T. Koyama, M. Tachiki, Phys. Rev. B 54 (1996) 16183.
[9] M. Machida, T. Koyama, M. Tachiki, Phys. Rev. Lett. 83 (1999) 4618.
[10] See, e.g. M. Machida, S. Sakai, Phys. Rev. B 70 (2004) 144520.
[11] T. Koyama, M. Machida, M. Kato, T. Ishida, Physica C 463–465 (2007) 985.
[12] M. Machida, T. Koyama, Phys. Rev. B 70 (2004) 024523.
[13] S. Yamada, T. Kano, T. Imamura, M. Machida, unpublished.
[14] See, e.g. S. Strogatz, Sync: The Emerging Science of Spontaneous Order, first ed., Hyperion, 2003.