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Methods for deriving a representative biodynamic response of the hand–arm system to vibration

Ren G. Dong *, Daniel E. Welcome, Thomas W. McDowell, John Z. Wu

National Institute for Occupational Safety and Health, Engineering & Control Technology Branch, 1095 Willowdale Road, Morgantown, WV 26505, USA

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A B S T R A C T

Vibration-induced biodynamic responses (BR) of the human hand–arm system measured with subjects participating in an experiment are usually arithmetically averaged and used to represent their mean response. The mean BR data reported from different studies are further arithmetically averaged to form the reference mean response for standardization and other applications. The objectives of this study are to clarify whether such a response-based averaging process could significantly misrepresent the characteristics of the original responses, and to identify an appropriate derivation method. The arithmetically averaged response was directly compared with the response derived from a property-based method proposed in this study. Two sets of reported mechanical impedance data measured at the fingers and the palms of the hands were used to derive the models required for the comparison. This study found that the response-based arithmetic averaging could generate some systematic errors. The range of the subjects' natural frequencies in each resonance mode, the mode damping ratio, and the number of subjects participating in the experiment are among the major factors influencing the level of the errors. An effective and practical approach for reducing the potential for error is to increase the number of subjects in the BR measurement. On the other hand, the property-based derivation method can be generally used to obtain the representative response, but it is less efficient than the response-based derivation method.

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1. Introduction

The vibration-induced biodynamic response (BR) of the human hand–arm system can be expressed in many forms such as vibration-induced stresses and strains, driving-point apparent mass and mechanical impedance, vibration power absorption, and vibration transmissibility [1]. These measures can be used for identifying the biodynamic characteristics of the hand–arm system, for helping to develop better tools and anti-vibration devices, for understanding vibration-induced psychophysical responses and health effects, and for helping to improve methods to assess the risks of hand-transmitted vibration exposure [1–3]. Like height and weight, the BR is individual-specific. The responses measured with the subjects participating in an experiment are usually averaged at each frequency with a simple arithmetic averaging method, and the results are used to represent their mean response [e.g., 4–11]. Several sets of the averaged driving-point mechanical impedance spectra of the hand–arm system were selected and further arithmetically averaged [12], and the results are recommended as the reference mean values in an international standard (ISO 10068, 1998) [3]. Computer models of the hand–arm system are also usually developed or validated based on the arithmetically averaged response [e.g., 3,13,14].

* Corresponding author. Tel.: +1 304 285 6332; fax: +1 304 285 6265.
E-mail address: rkd6@cdc.gov (R.G. Dong).

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However, it remains an issue whether these mean values are the representative biodynamic response of the hand–arm system.

Some research has demonstrated that arithmetic averaging can reduce the resonant peak and change the representation of frequency-dependant characteristics of the biodynamic response of the whole-body system [15,16]. Similar effects have also been observed in the biodynamic response of the hand–arm system [11,17]. Whereas these observations cast some doubt on this common derivation method, its suitability and limitations have not been sufficiently examined. It is also unclear how to analyze potential systematic errors that could result from the response-based arithmetic averaging process. Whereas a modal description method for deriving the representative whole-body biodynamic response has been proposed [15], an alternative method for deriving the representative biodynamic responses distributed at the fingers and the palm of the hand has not been reported. Considering that the vast majority of the reported data and models of the hand–arm system were derived using the response-based averaging method, it seems necessary to assess the impact of the arithmetic averaging effects on their validity and further applications. It is also important to conduct a systematic analysis of derivation methodologies for further studies of the biodynamic responses, the revision of the reference values in the ISO standard, and further developments of hand–arm system models.

Although the specific effects of the arithmetic averaging process on the derivation of the mean biodynamic response of the hand–arm system have not been quantified, its general effects can be qualitatively understood by examining the comparisons shown in Fig. 1. The comparisons reveal several basic characteristics of the averaging effects, which are summarized as follows: (1) the larger the distance is between any two peak frequencies, the more their averaged peak value is reduced (compare Fig. 1(a) with Fig. 1(b)); (2) the sharper the peaks are, the larger the effect of averaging (compare Fig. 1(a) with Fig. 1(c)); (3) the averaging process could distort the dynamic characteristics such as increasing the number of peaks and changing the shape of the response peak, as shown in Fig. 1(a) and Fig. 1(c); and (4) increasing the number of subjects will lessen the averaging effects if the range of the peak frequencies remains unchanged (compare Fig. 1(a) with Fig. 1(d)).

Based on these observations, this study made the following hypotheses: (i) because the resonant frequencies of the subjects participating in an experiment are usually different, the arithmetic averaging process could introduce some errors in deriving the representative biodynamic response; (ii) because the hand–arm system is usually a heavily damped structure, the arithmetic averaging effects are not dramatic; (iii) the averaging effects depend on the specific biodynamic characteristics of the subjects; and (iv) the averaging effects on the BR can be reduced to an acceptable level if the number of subjects participating in the study is sufficiently large. Based on these hypotheses, this study further hypothesized that the response-based arithmetic average could be problematic in some cases, but it could be acceptable if it is properly applied under certain conditions.

This study tested these hypotheses by quantifying the arithmetic averaging effects. A systematic property-based derivation method was proposed and used to derive the reference response for the quantification. The objective of this study was to determine how to best derive the representative biodynamic response of the hand–arm system to vibration.

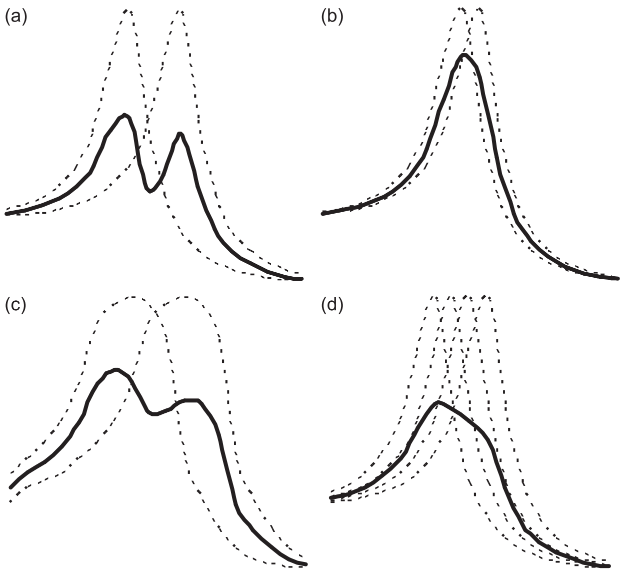

Fig. 1. Factors affecting the arithmetic averaging effects: (a) comparison of the original responses and the averaged response; (b) effect of the distance between peaks or natural frequencies; (c) effect of the damping of the system on the kurtosis of the response; and (d) effect of the number of subjects (------- assumed original data; ▬▬▬▬ averaged data).

2. Method

2.1. Basic concept and approach

To test the hypotheses of this study, it is necessary to quantify the effects of averaging; this can be done by comparing the 'distorted' average response with a baseline average response. One critical issue is how to define and derive the baseline average response. This study defines the baseline average response as the response of a virtual subject who exhibits the average biodynamic properties (the mass, damping, stiffness, and their distributions and connections) of all subjects who participated in the experiment. Because the biodynamic properties are of major concern, the baseline method based on this definition is termed as the property-based derivation method to differentiate it from the conventional biodynamic response-based derivation method. The differences between the arithmetic averaging response and the mean property-based response are thus considered as the errors of the response-based derivation method.

To construct the virtual mean subject, it is necessary to quantify the biodynamic properties of the subjects participating in a study. So far, a feasible technology has not been developed to directly measure the required biodynamic properties of the hand–arm system of a subject. Alternatively, this study proposes a modeling approach to estimate the biodynamic properties as follows: (I) a model structure that provides a reasonable simulation of the hand–arm system is selected or configured and (II) each subject's model parameters are determined using their individual biodynamic responses measured in an experiment.

The basic model structure is assumed to be the same for each subject. However, each subject exhibits unique values for each component of the model. Once each subject's individual model parameters are determined, the model for the virtual mean subject is constructed by calculating the arithmetic mean values for each model component. The virtual subject model is then used to generate the mean property-based BR.

On the other hand, each individual's unique model is used to generate a BR value for each subject. The average of these BR values is called the mean response-based BR. According to the above definition, the difference between the mean property-based BR and the mean response-based BR is considered to be a systematic error induced by the response-based method.

It is emphasized that in this study, the BR values measured in the laboratory are not directly used to evaluate the effects of the response-based averaging method. This is because the difference between the measured BR mean and the mean property-based BR would result from both the arithmetical averaging errors and the modeling residuals. In contrast, the difference between the mean response-based BR and the mean property-based BR calculated from the models would result solely from the effects of the response-based averaging method; modeling residuals would not contribute to the error because the same models are used to generate both BR means.

2.2. Identifications of individual biodynamic properties

The model structure of the hand–arm system used in this study is shown in Fig. 2, which was originally reported in Ref. [14]. This model has only three effective DOF. The modeling can be conducted using widely distributed commercial software (e.g., Matlab or MS Excel), and the model parameters can be identified using a simple optimization procedure [14,18]. The reported results demonstrate that this model can fit the mean responses of the subjects of several experiments well [e.g., 14,18]. As found in the current study, this model can also reasonably fit the distributed responses of each individual.

Two sets of the laboratory-measured data reported from previous studies were used in the current study. The first set of data is shown in Fig. 3, together with their arithmetically averaged spectra. They were measured with six male subjects along their forearm direction (z_h-axis) [19]. Each subject was required to use the hand and arm postures and coupling forces (30 N grip and 50 N push) standardized for testing anti-vibration gloves [20]. A broad-band random vibration spectrum was used as the excitation in the measurement. These testing conditions, except the hand coupling forces, were also used in the measurements of the second set of data used in the current study. They were reported from two studies that separately measured the biodynamic responses distributed at the fingers and the palm of the hand [11,17]. Eight male subjects participated in both studies. Their impedance responses measured under a combined 50 N grip and 50 N push were thus selected for the current study, which are plotted in Fig. 4. These mechanical impedances were expressed in the one-third octave bands from 10 to 1000 Hz. The anthropometric measures of these two groups of subjects are listed in Table 1.

The impedance spectra of each subject were used to determine the parameters of the model for that subject using the procedures developed in previous studies [14,18]. Briefly, the equations of motions of the model shown in Fig. 2 were written and resolved to derive the motions of the five mass elements and the forces acting on each connecting element for a given handle acceleration input to the model. The mechanical impedances distributed at the fingers and the palm driving points were calculated using the derived motions and forces. The difference between the modeling impedance and the laboratory-measured impedance at each driving point at each frequency was calculated. The summation of the root-mean-square (rms) values of the differences in the real and imaginary parts of the impedances was used as an error function. The error function was minimized to achieve the optimized parameters of the model with the same constraints as used in

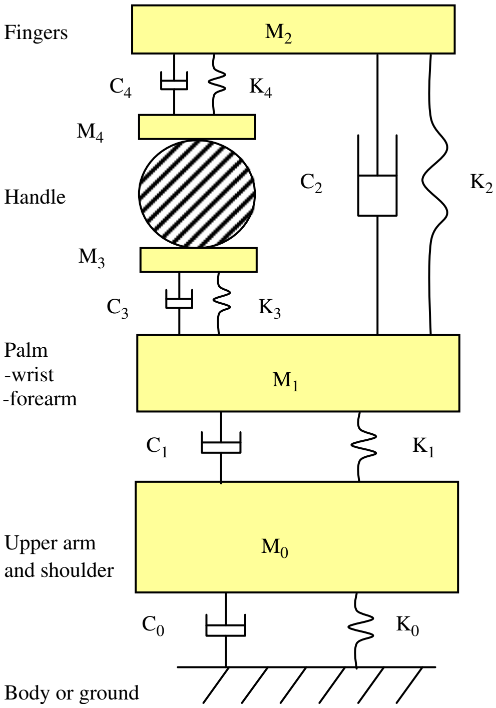

Fig. 2. A 5-DOF model of the hand–arm system [14].

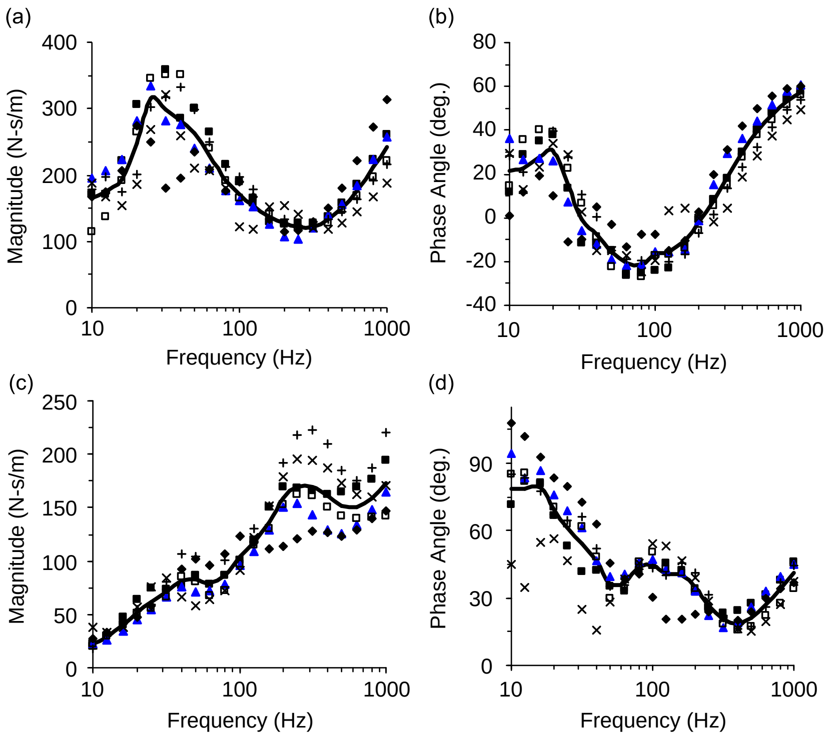

Fig. 3. Driving-point mechanical impedances of the hand–arm system measured with six male subjects [19]: (a) palm magnitude; (b) palm phase angle; (c) fingers magnitude; and (d) fingers phase angle (■ Subject 1; ▲ Subject 2; □ Subject 3; × Subject 4; ◆ Subject 5; +Subject 6; ——— arithmetic mean).

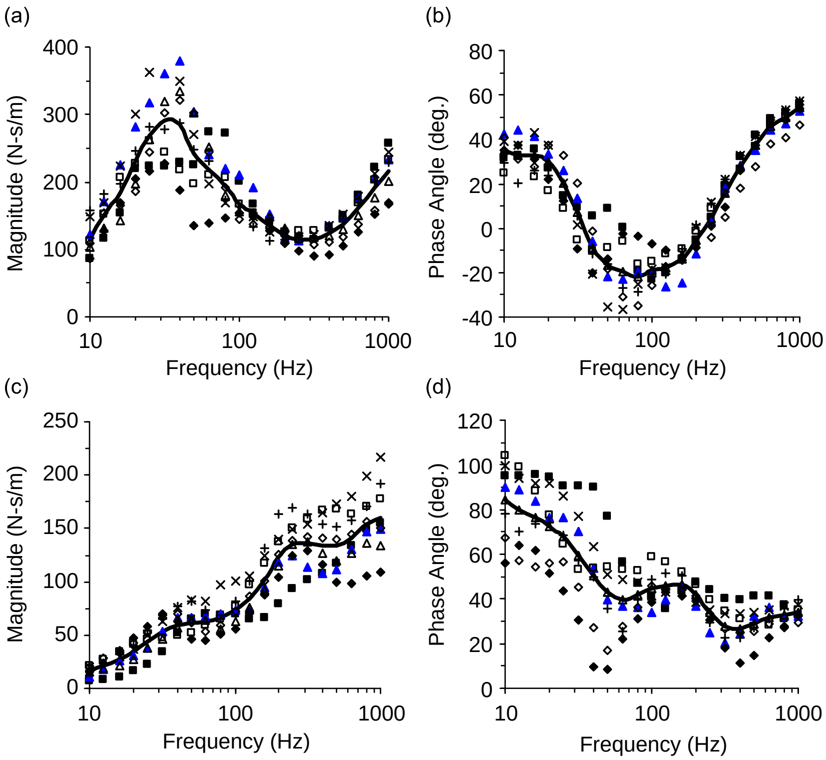

Fig. 4. Driving-point mechanical impedances of the hand–arm system measured with eight male subjects [11,17]: (a) palm magnitude; (b) palm phase angle; (c) fingers magnitude; and (d) fingers phase angle (■ Subject 1; ▲ Subject 2; □ Subject 3; × Subject 4; ◆ Subject 5; +Subject 6; △ Subject 7; ◇ Subject 8; ▬▬ arithmetic mean).

Table 1
Anthropometry of the subjects participating in the experiments (hand length = tip of middle finger to crease at the wrist; hand breadth = the width measured at the metacarpals; and hand circumference = the circumference measured at the metacarpals).

Subject	Height (cm)	Weight (kg)	Hand length (mm)	Hand breadth (mm)	Hand circumference (mm)
The six male subjects in the first set of data [19]					
1	182.9	88.4	192	89	215
2	182.9	97.5	196	91	220
3	177.8	77.2	193	83	206
4	185.0	104.0	191	92	222
5	177.8	104.0	186	89	210
6	175.0	75.4	192	87	210
Mean	180.2	91.1	192	89	214
Std	3.9	12.8	3	3	6
CV	0.022	0.141	0.017	0.036	0.029
The eight male subjects in the second set of data [11,17]					
1	175.3	69.5	185	88	
2	177.8	83.0	197	93	
3	185.4	90.7	192	97	
4	175.3	132.5	207	101	
5	175.3	100.2	184	103	
6	185.4	66.2	197	93	
7	185.4	96.6	200	101	
8	175.3	77.1	190	85	
Mean	179.4	89.5	194	95	
Std	5.1	21.2	8	7	
CV	0.028	0.237	0.041	0.074	

previous studies [18,19], which are as follows:

$$M_0, M_1, M_2, M_3, M_4, K_0, K_1, K_2, K_3, K_4, C_0, C_1, C_2, C_3, C_4 > 0$$
$$M_0 < 15\,\text{kg (shoulder and a part of the upper body)}$$

$$\begin{aligned}&M_1 < 5\,\text{kg (palm, hand back, wrist and forearm)}\\&M_2 < 200\,\text{g (fingers bones and part of the finger soft tissues)}\\&M_3 < 50\,\text{g (palm contact skin)}\\&M_4 < 30\,\text{g (finger contact skin).}\end{aligned} \tag{1}$$

2.3. Property-based mean virtual subject model

After the models for each of the subjects were constructed, each parameter (P_{i_Mean}) of the property-based mean virtual subject model was calculated from

$$P_{i\_\mathrm{Mean}} = \frac{1}{n}\sum_{k=1}^{n} P_{ik}, \tag{2}$$

where n is the number of subjects considered in the average, P_i is the ith corresponding parameter value, and k denotes the kth subject.

2.4. Calculation of the biodynamic response arithmetic averaging error

The model for each subject was used to calculate the modeling response (Z_j) of the subject at each (ω) of the one-sixth octave band frequencies from 10 to 1000 Hz. They were used to derive the response-based mean response ($Z_{\mathrm{Mean\_response}}$) using the conventional arithmetic complex value average method:

$$Z_{\mathrm{Mean\_response}}(\omega) = \frac{1}{n}\left\{\sum_{k=1}^{n} Z_{k\_\mathrm{Real}}(\omega) + \mathrm{j}\sum_{k=1}^{n} Z_{k\_\mathrm{Imaginary}}(\omega)\right\}. \tag{3}$$

The virtual subject model established with the parameters evaluated from Eq. (2) is used to calculate the mean property-based response. The difference (ΔZ) between these two types of average responses was calculated from

$$\Delta Z(\omega) = |Z_{\mathrm{Mean\_property}}(\omega) - Z_{\mathrm{Mean\_response}}(\omega)|. \tag{4}$$

The difference was considered as the error by which to judge the effects of the response-based averaging method. To examine the influence of the number of subjects on the response-based averaging errors, all possible combinations of 2, 3, 4, 5, and 6 subjects among the six subjects in the first set of data and of 2, 4, 6, and 8 subjects among the eight subjects in the second set of data were considered in this study.

2.5. Additional comparison

As above-mentioned, the mean response of the laboratory-measured data is conventionally used to conduct the modeling and to derive the parameters of the mechanical equivalent model [e.g., 12–14]. The modeling results obtained from this approach were also compared with those obtained from the above-mentioned mean property-based method.

2.6. Estimations of natural frequencies and damping ratios

To help understand the modeling results and the arithmetic averaging effects, the undamped natural frequencies and critical damping ratios of each model were calculated by performing an eigenvalue analysis. For this purpose, it is not critical to obtain the accurate damping ratios. Therefore, for simplicity and as a crude approximation, each critical damping ratio (ξ) was estimated using the undamped eigenvalue or natural frequency and its corresponding eigenvector [21]. The diagonal elements of the normalized matrices were used to estimate the critical damping ratios.

3. Results

3.1. Models

As examples, Fig. 5 shows typical comparisons of the modeling results for two subjects (Subject 4 from each set of the data) with their experimental data measured at the fingers and the palm of the hand of these subjects. As anticipated, the modeling responses do not match every detail of the measured responses. The modeling response spectra are generally smoother than the laboratory-measured response spectra. This suggests that there are more vibration modes than what were considered in the modeling. However, the dominant vibration modes are well reflected in the modeling response spectra. The model generally fits the laboratory-measured data very well. This is also evidenced from the low mean residual rms value (11.5 N s/m for the first set of data and 8.8 N s/m for the second set of data) and the high mean r^2-value

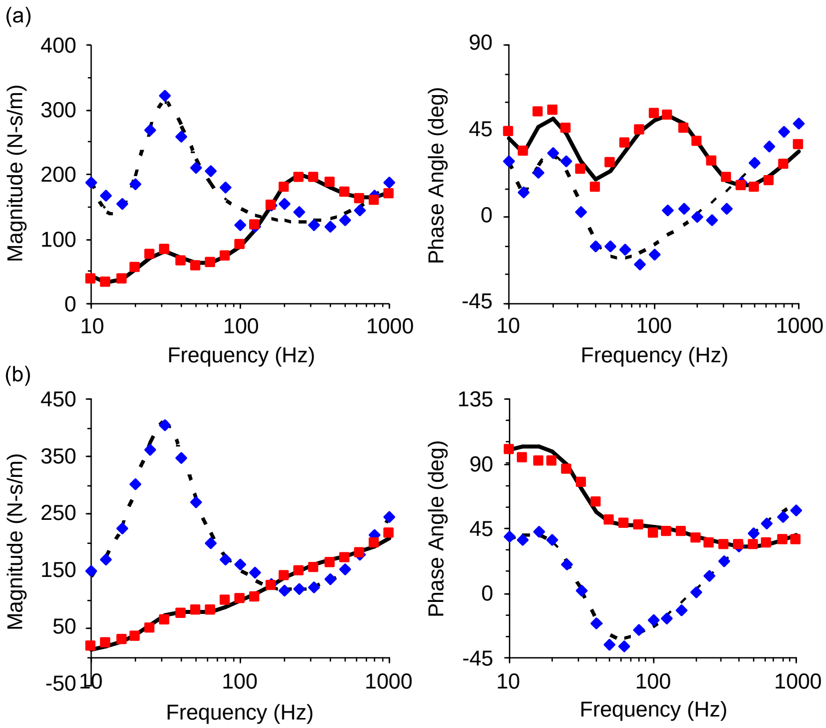

Fig. 5. Comparisons of driving-point mechanical impedance magnitude and phase, derived from the hand–arm model, with the laboratory-measured data measured with 2 subjects: (a) Subject 4 from the six-subject data [19] and (b) Subject 4 from the eight-subject data [11,17] (◆ palm experiment; ------- palm model; ■ fingers experiment; ——— fingers model).

(0.95 for the six-subject data and 0.98 for the eight-subject data). These observations suggest that the derived individual models provide a reasonable representation of the major biodynamic properties of these subjects' hand–arm systems for the given experimental conditions.

Fig. 6 shows the comparison of the laboratory-measured mean BR and the modeling response derived from the laboratory-measured mean BR. These modeling responses have lower mean residual rms values (8.0 N s/m for the six-subject data and 5.7 N s/m for the eight-subject data) and higher r^2-values (0.98 for the six-subject data and 0.99 for the eight-subject data) than those of the individual models mentioned in the last paragraph. These observations demonstrate that on the average, the model derived from the laboratory-measured mean response generally fits the laboratory-measured data better than the individual subject models do.

The identified model parameter values for the subjects are listed in Tables 2 and 3, together with their mean values (or property-based average model) and the parameter values of the laboratory-measured mean response-based model (in the last column of the table). A large coefficient of variation (CV >0.15) of the parameter values was observed among the subjects. This is consistent with the large variations of the laboratory-measured biodynamic responses shown in Figs. 3 and 4.

The undamped natural frequencies and the estimated critical damping ratios are also listed in Tables 2 and 3. The second frequency is generally very close to the highest peak magnitude of the palm response shown in Figs. 3(a) and 4(a). Because the frequency estimated from $\sqrt{K_3/M_1}/2\pi$ is very close to this frequency, this resonance is obviously related to the effective mass (M_1) of the palm-wrist-forearm structures and the palm contact stiffness (K_3). The damping ratio estimated from $C_3/(2\sqrt{M_1K_3})$ is much less than the second critical damping ratio listed in the tables, but it can be closely estimated from $(C_1+C_3)/[2\sqrt{M_1(K_1+K_3)}]$. This suggests that the major energy in this resonance region is dissipated not only in the palm but also in the wrist-arm system. For each model, the first frequency is very close to that estimated from $\sqrt{(K_0+K_1)/M_0}/2\pi$, and the first critical damping ratio is very close to that estimated from $(C_0+C_1)/[2\sqrt{M_0(K_0+K_1)}]$. These relationships suggest that the first vibration mode is mainly associated with the vibration motions of the upper arm and shoulder structures. The third natural frequency mainly depends on the finger effective mass and the contact stiffness, as can be estimated from $\sqrt{K_4/M_2}/2\pi$. This frequency is also in the neighborhood of the fingers' peak response, as shown in Figs. 3 and 4. Our previous study also found that the variation of the palm contact force has little influence on the response of the fingers in this resonance region [11]. Therefore, this vibration mode is mainly associated with the fingers' major resonance.

As it can be seen in the last two columns of Tables 2 and 3, the differences between the paired parameter values of the property-based average model and the model derived from the conventional method (i.e., determining the model

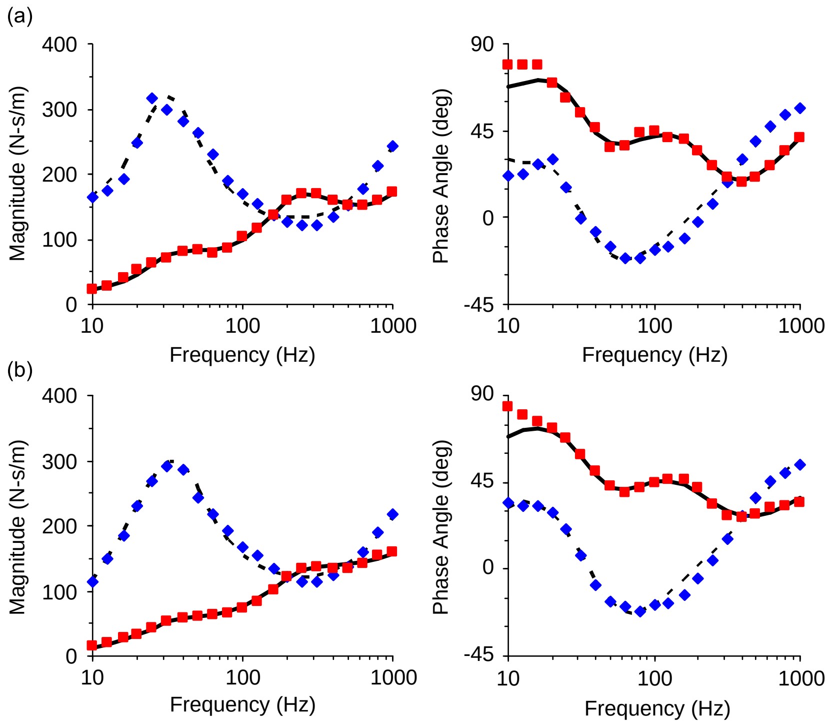

Fig. 6. Comparisons of the laboratory-measured mean responses and the modeling responses calculated using the conventional modeling method [14,18,19]: (a) with the six-subject data and (b) with the eight-subject data (◆ palm laboratory-measured mean response; ------- palm modeling response; ■ fingers laboratory-measured mean response; ▬▬▬ fingers modeling response).

Table 2
Parameters of the hand–arm system models for the six-subject data [19] (f: the undamped natural frequency and ξ: the estimated critical damping ratio).

Parameter	Unit	Model for each subject						Mean property model	Mean response model
		1	2	3	4	5	6		
M_0	kg	4.731	5.228	4.837	6.803	7.500	6.884	5.997	6.015
M_1	kg	1.747	1.473	1.397	1.675	0.799	1.401	1.416	1.462
M_2	kg	0.110	0.090	0.095	0.112	0.091	0.101	0.100	0.096
M_3	kg	0.036	0.037	0.032	0.023	0.047	0.029	0.034	0.034
M_4	kg	0.022	0.020	0.014	0.016	0.013	0.026	0.019	0.019
K_0	N/m	8053	6614	8118	18 352	6476	15 654	10 545	7567
K_1	N/m	2987	2316	1844	7198	1316	4839	3417	2978
K_2	N/m	5524	1997	5252	8935	2145	6639	5082	4221
K_3	N/m	59 949	51 690	64 940	43 125	76 811	67 824	60 723	55 564
K_4	N/m	166 181	158 582	179 694	208 936	108 660	296 962	186 503	196 038
C_0	N s/m	399	32	397	45	275	41	199	106
C_1	N s/m	163	135	138	90	229	125	147	134
C_2	N s/m	48	44	44	37	90	64	54	52
C_3	N s/m	132	118	118	123	119	127	123	126
C_4	N s/m	132	101	117	136	119	131	123	122
f_1	Hz	7.6	6.5	7.2	9.6	5.1	8.6	7.6	6.6
f_2	Hz	31.5	31.0	36.1	29.9	50.4	37.8	35.2	33.0
f_3	Hz	198.7	212.2	221.6	222.4	175.2	275.7	220.3	230.1
ξ_1		1.24	0.41	1.23	0.19	1.05	0.24	0.61	0.49
ξ_2		0.48	0.51	0.47	0.38	0.86	0.47	0.51	0.51
ξ_3		0.65	0.60	0.60	0.56	1.04	0.56	0.64	0.63

parameter values using the laboratory-measured mean BR) are surprisingly small ($<5\%$) in the vast majority of the paired comparisons. Their undamped natural frequencies and critical damping ratios are also very similar, except those for the first resonance.

Table 3
Parameters of the hand–arm system models for the eight-subject data [11,17] (f: the undamped natural frequency and ξ: the estimated critical damping ratio).

Parameter	Unit	Model for each subject								Mean property model	Mean response model
		1	2	3	4	5	6	7	8		
M_0	kg	5.208	3.064	5.825	3.000	6.352	6.393	3.000	5.412	4.782	4.670
M_1	kg	0.615	1.178	0.793	1.491	1.361	1.182	1.162	1.234	1.127	1.142
M_2	kg	0.037	0.074	0.081	0.114	0.061	0.092	0.066	0.090	0.077	0.079
M_3	kg	0.037	0.032	0.032	0.035	0.022	0.033	0.027	0.022	0.030	0.030
M_4	kg	0.007	0.013	0.014	0.018	0.011	0.020	0.011	0.011	0.013	0.013
K_0	N/m	20 972	8258	22 154	7325	29 153	4946	10 863	16 381	15 006	14 596
K_1	N/m	1340	1082	1000	1047	2090	1771	2412	4663	1926	1463
K_2	N/m	710	3462	2909	0	5651	5284	2487	9935	3805	3710
K_3	N/m	101 341	74 855	63 992	59 864	31 809	60 396	63 724	60 502	64 560	58 351
K_4	N/m	236 994	116 079	179 787	87 392	168 025	187 399	166 824	126 025	158 566	137 739
C_0	N s/m	126	136	164	309	235	205	400	86	208	222
C_1	N s/m	165	174	204	126	94	160	114	111	144	142
C_2	N s/m	48	40	34	36	37	42	32	31	38	36
C_3	N s/m	109	123	123	111	101	107	113	108	112	116
C_4	N s/m	160	109	144	161	79	125	106	130	127	124
f_1	Hz	10.4	8.8	10.0	8.4	11.1	5.1	10.6	9.9	9.5	9.2
f_2	Hz	65.2	41.3	46.5	32.2	27.1	38.0	38.7	39.1	39.5	37.5
f_3	Hz	400.7	202.7	239.1	139.3	269.2	230.8	254.7	195.4	231.3	209.5
ξ_1		0.43	0.93	0.51	1.38	0.38	0.89	1.30	0.31	0.52	0.75
ξ_2		0.63	0.54	0.78	0.45	0.49	0.54	0.45	0.40	0.51	0.53
ξ_3		1.10	0.80	0.73	0.99	0.57	0.63	0.65	0.73	0.74	0.77

3.2. Response-based averaging effects

As shown in Tables 2 and 3, the second natural frequencies of all the subjects are within the range of 27.1 (for Subject 5 in Table 3) to 65.2 Hz (for Subject 1 in Table 3). Corresponding to this maximum frequency difference, the mean response-based BR derived from the modeling responses of the two boundary subjects shows the largest difference from that derived from the property-based method. As shown in Fig. 7(a), response-based averaging greatly reduced the resonant peak in this case. Although the second frequency is mainly associated with the response distributed at the palm, the largest response-based averaging effect was also found in the derived fingers response for the combination of these two subjects. The maximum magnitude errors in the frequency range of 16–1000 Hz for all possible combinations of two subjects in each set of data were evaluated. As shown in Fig. 8, the difference between the second natural frequencies of the two subjects is reliably correlated with the maximum percent magnitude error of the response-based BR ($r^2 \geqslant 0.753$, $p < 0.001$). Because the second natural frequency range of the first set of data ($\Delta f_{2\_\mathrm{Max}} = 50.4–29.9$ Hz) is much smaller than that of the second set of data ($\Delta f_{2\_\mathrm{Max}} = 65.2–27.1$ Hz), the maximum errors from the first set of data (Fig. 8(c) and (d)) are obviously smaller than those from the second set of data (Fig. 8(a) and (b)). As also shown in Fig. 8, the maximum errors are less than 10% when the frequency difference is less than 10 Hz in each case.

When the combinations of four subjects were considered in the derivation of the responses, the maximum difference was found from the combination of Subjects 1, 3, 4 and 5 listed in Table 3. As shown in Fig. 7(b), the differences between the two types of responses are less than those shown in Fig. 7(a). When all eight subjects in Table 3 were considered, the modeling responses derived from these two methods are very similar, as shown in Fig. 7(c). Similar phenomena were also observed in the derived responses of the six subjects listed in Table 2.

After the difference between the responses derived from the two methods at each one-sixth octave band frequency for every possible combination of a given number of subjects was calculated using Eq. (4), the maximum value among the differences for each number of subjects were identified. The resulting maximum difference spectra for the first group of subjects are plotted in Fig. 9. Obviously, the maximum difference generally increases with the reduction of the number of subjects for both the finger and palm responses expressed in both magnitude and phase angle. At the palm side, discernible differences are mainly restricted to frequencies below 100 Hz; this is because the resonances of the palm–wrist–arm system usually occur below this frequency. The response at the fingers was affected by the response-based averaging process in a much larger frequency range; the finger response is influenced not only by the global resonances of the entire hand–arm system but also by the local finger resonances [11,14,18].

As shown in Fig. 9(e) and (f), the percent maximum difference in each resonant range seems associated with both the critical damping ratio (ξ) shown in Table 2 and the relative frequency ratio (β) defined as follows:

$$\beta_i = \frac{f_{i\_\mathrm{Max}} - f_{i\_\mathrm{Min}}}{(f_{i\_\mathrm{Max}} + f_{i\_\mathrm{Min}})/2}, \tag{5}$$

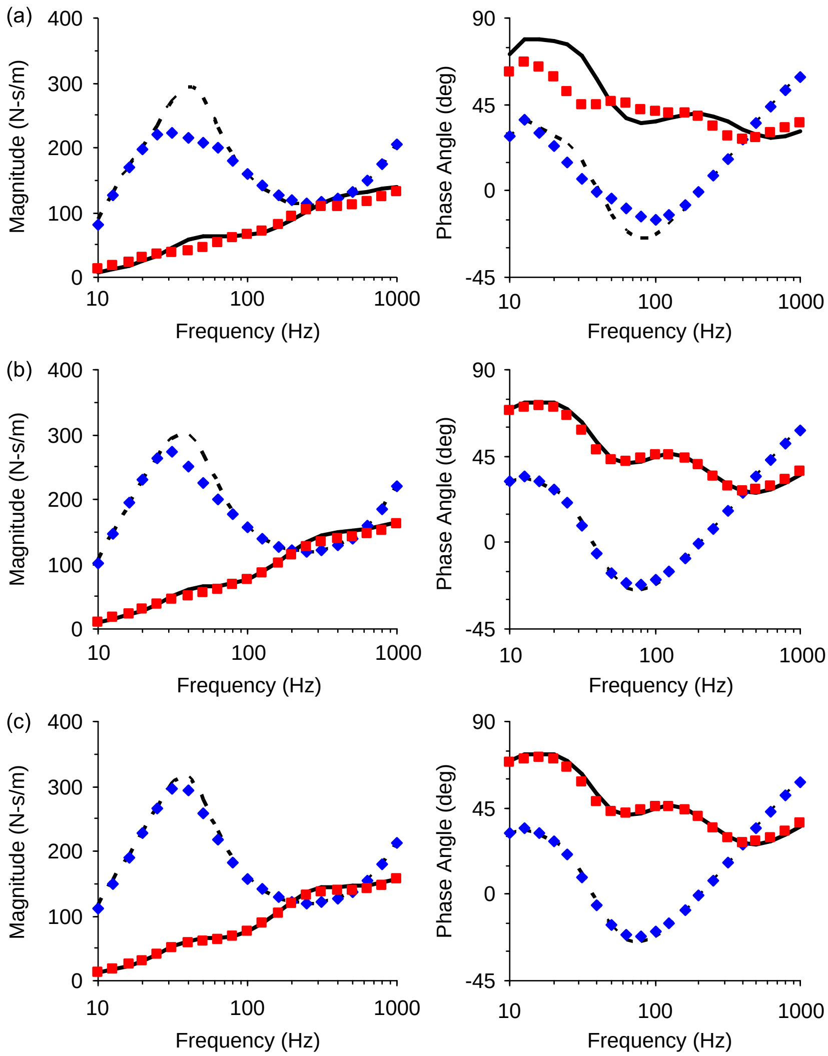

Fig. 7. Examples of arithmetic averaging effects on the conventional synthesis of the hand–arm biodynamic response: (a) derivation with two subjects (Subjects 1 and 5 in Table 3); (b) derivation with four subjects (Subjects 1, 3, 4 and 5 in Table 3); and (c) derivation with all the eight subjects in Table 3. (◆ palm response-based deriving method; ------- palm property-based deriving method; ■ fingers response-based deriving method; ▬▬ fingers property-based deriving method).

where f_{i_Max} and f_{i_Min} are the ith mode maximum and minimum frequencies, respectively, among all of the subjects. With this equation, the frequency ratios ($\beta_1 = 0.61$, $\beta_2 = 0.51$, and $\beta_3 = 0.45$) of the three resonance modes were calculated from the resonant frequencies listed in Table 2. Because ξ_3 is greater than or equal to 0.56, and β_3 is the smallest frequency ratio, the lowest response-based averaging effect occurs in the third resonant frequency range (>100 Hz), as shown in Fig. 9. Because ξ_1 ($\geqslant 0.19$) includes the lowest damping value and β_1 is the highest frequency ratio, the largest percentage differences for both the fingers and palm responses occur at frequencies below 12.5 Hz. The maximum percentage difference is greater than 30% at frequencies below 10 Hz, which is not plotted in these figures. These observations also support the hypotheses of this study.

The maximum difference spectra for the second set of data (eight-subject data) were plotted in Fig. 10. The basic effects of the subject number on the differences are similar to those observed in Fig. 9. The response-based averaging-induced errors in the palm response are also primarily distributed at frequencies less than 100 Hz and the response-based averaging errors in the fingers response are also distributed in a larger frequency range. Because the frequency ratios ($\beta_1 = 0.74$, $\beta_2 = 0.83$, $\beta_3 = 0.97$) for this group of subjects are larger than those for the first group of subjects, the response differences for the same number of subjects shown in Fig. 10 are generally larger than those shown in Fig. 9.

However, in the major frequency range of concern (25–500 Hz) for hand–arm vibration syndrome [22], the percent differences between the response-based averaging and the property-based averaging are not substantial when six or more subjects are considered in the response derivations. In such cases, the maximum error induced from response-based averaging is less than 15%, as shown in Figs. 9 and 10. Such differences are comparable with the average intra-subject

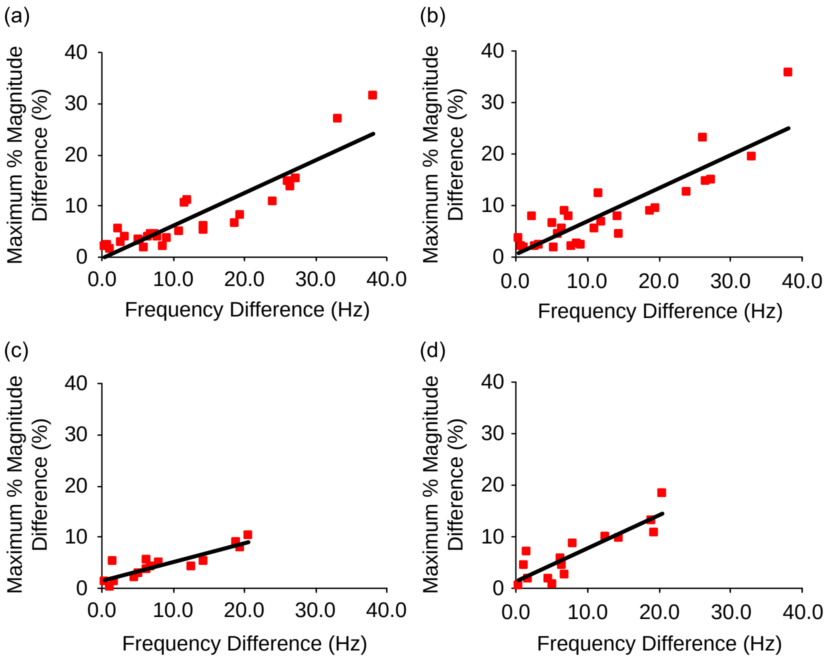

Fig. 8. The relationship between the second natural frequency difference ($\Delta f = f_{2\_\mathrm{Subject}\_i} - f_{2\_\mathrm{Subject}\_j}$, $i \neq j$) of two subjects and the maximum error of the arithmetically averaged response in the frequency range of 16 to 1000 Hz: (a) palm maximum percent magnitude error from the eight-subject data; (b) fingers maximum percent magnitude error from the eight-subject data; (c) palm maximum percent magnitude error from the six-subject data; and (d) fingers maximum percent magnitude error from the six-subject data. (■ the relationship raw data; ▬▬ trendline).

variations observed in the reported experiments [e.g., 11,17,19]. For example, the maximum mean difference for impedance magnitude measured at the palm is about 14.5% in Ref. [19], and the corresponding maximum for the fingers is about 16.8%. When five or more subjects in the first set of data are considered in the derivation of the representative biodynamic response using the response-based method, the maximum response-based averaging error is less than 10%, as shown in Fig. 9(e) and (f). It is also less than 10% in the second set of data when seven or more subjects are considered, as shown in Fig. 10(e) and (f).

4. Discussion

The arithmetically averaged biodynamic response of the hand–arm system is usually reported and/or used to represent the population response of the subjects participating in a study. The current study evaluated this practice through the comparison of the mean response-based BR and the BRs derived from a property-based averaging method proposed in this study. The results can be used to identify and understand their differences and similarities and to apply them appropriately.

4.1. Response-based derivation method

The conventional arithmetical method is simple to use, and the resulting mean response is unique. Furthermore, the modeling of the biodynamic response is not required in some applications such as the derivation of the biodynamic frequency weighting of the entire hand–arm system [23] and the examination of the relationships between the biodynamic response and discomfort and health effects [24]. For such applications, the response-based derivation method is certainly the first choice for deriving the representative response.

As confirmed in this study, the response-based derivation method could produce some systematic errors such as reducing the major resonant peak and modifying the shape of the original response spectrum, as shown in Fig. 7. The results of this study also indicate that the magnitudes of the errors induced by response-based averaging depend on the number of subjects participating in the measurement experiment, the range of the subjects' resonant frequencies in each vibration mode, and the damping ratio of each resonance. Wherein the resonant frequencies and damping ratios are natural properties of the subjects, and they cannot be changed, the use of a sufficient number of subjects in the measurement is a practical approach to control the response-based averaging effects to an acceptable level.

As shown in Figs. 9 and 10, the most significant errors induced by response-based averaging in the major frequency range of concern (25–500 Hz) for hand–arm vibration syndrome are primarily related to the second resonance or the resonance associated with the palm contact stiffness and the effective mass of the palm–wrist–forearm substructures. Therefore, the number of subjects required for an experiment depends mainly on the range of the second natural frequency

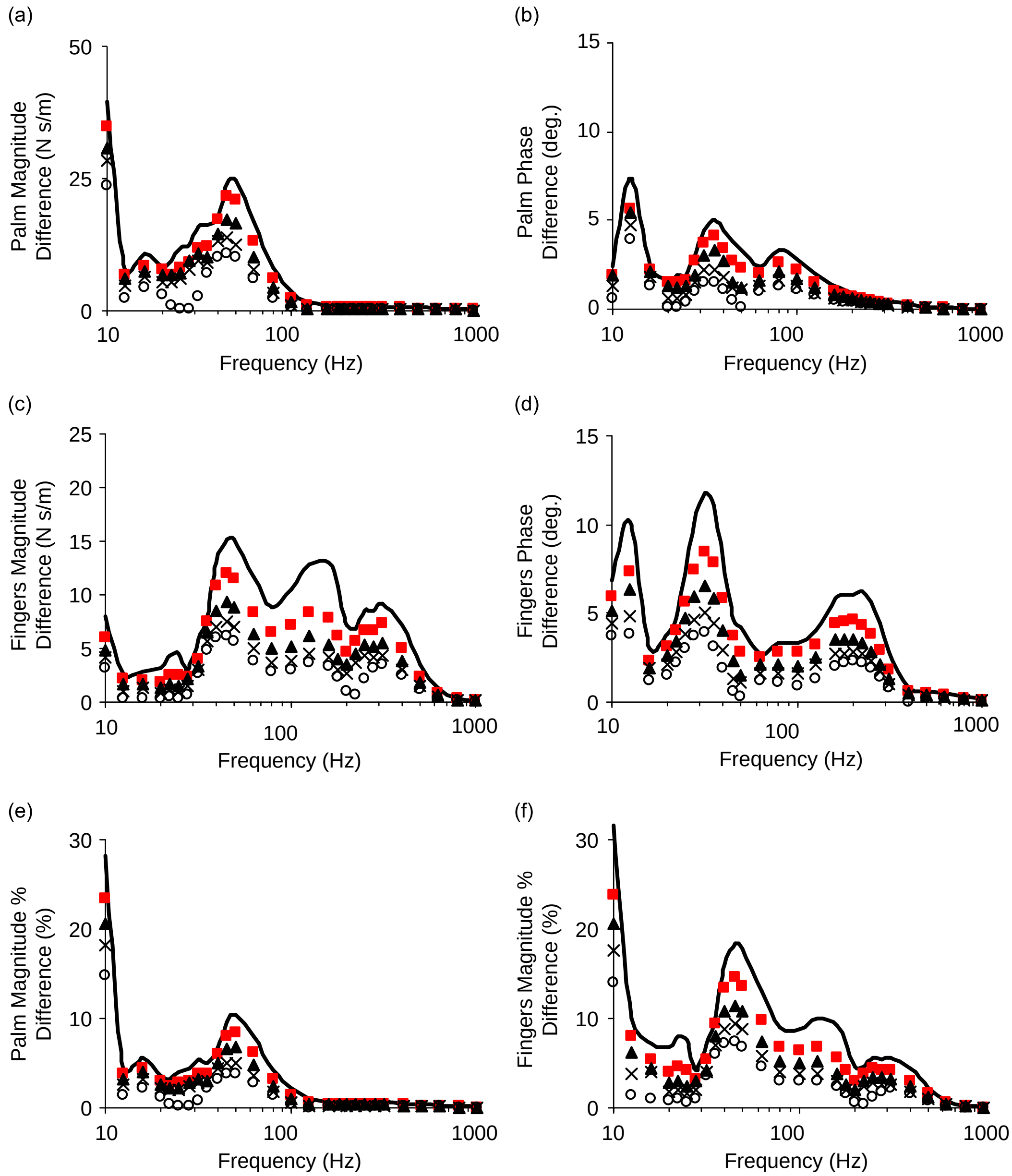

Fig. 9. One-sixth octave band distributions of the maximum difference between the response-based modeling responses and the property-based modeling responses among all the possible combinations for two, three, four, five, and six subjects among the six subjects in the six-subject data: (a) palm magnitude difference; (b) palm phase angle difference; (c) fingers magnitude difference; (d) fingers phase angle difference; (e) palm magnitude percent difference; and (f) fingers magnitude percent difference (◦ 6 subjects; × 5 subjects; ▲ 4 subjects; ■ 3 subjects; ▬▬ 2 subjects).

values. For example, two or more subjects are generally sufficient to control the maximum error at less than 10% for the response along the forearm direction if the maximum difference among the second natural frequencies of these subjects is less than 10 Hz, as shown in Fig. 8. For the same error level, five or more subjects are sufficient if the second relative resonant frequency ratio defined in Eq. (5) is less than 0.5, as shown in Fig. 9. When the relative frequency ratio is near 0.8, six or more subjects are required to control the error at the same level, as shown in Fig. 10. To control errors to levels below 5%, more subjects may be required for each case. These observed relationships may be applied to approximately assess the level of the potential errors or the sufficiency of the number of subjects in a study when the biodynamic response spectra of the subjects are available.

One may argue that adding one or more subjects in the experiment could also increase the width of the resonant frequency range and could thus increase the error induced by the response-based averaging. Therefore, it is possible that an increased number of subjects may not always be helpful. However, the resonant frequency of a particular vibration mode in a certain population that could be considered in a study is likely to be distributed in a certain range. Statistically, the added

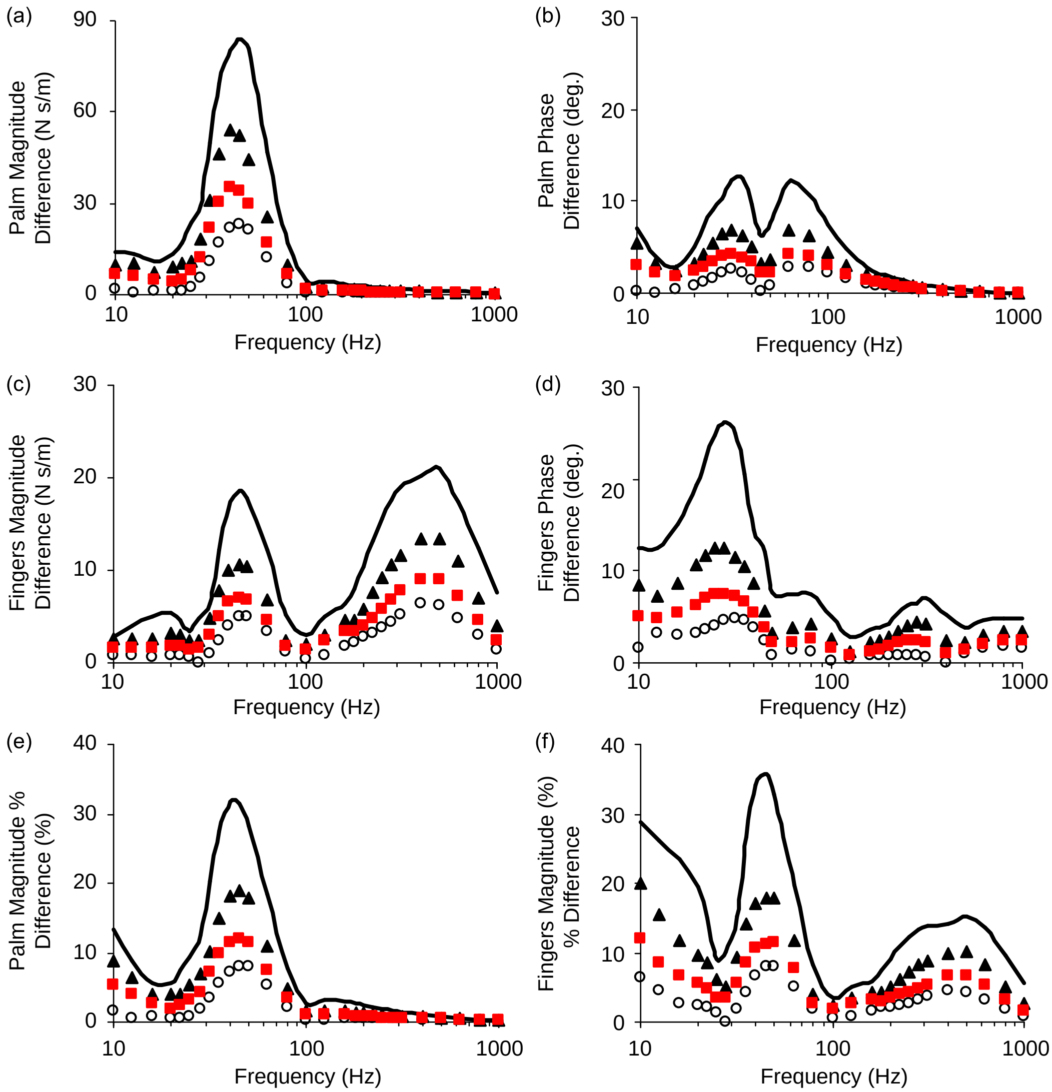

Fig. 10. One-sixth octave band distributions of the maximum difference between the response-based modeling responses and the property-based modeling responses among all the possible combinations for two, four, six, and eight subjects among the eight subjects in the eight-subject data: (a) palm magnitude difference; (b) palm phase angle difference; (c) fingers magnitude difference; (d) fingers phase angle difference; (e) palm magnitude percent difference; and (f) fingers magnitude percent difference (◦ 8 subjects; ■ 6 subjects; ▲ 4 subjects; ▬▬ 2 subjects).

subjects are unlikely to further increase the frequency range when the number of subjects reaches a certain point. If subjects with similar anthropometry could be grouped in the experiment, it is anticipated that the frequency range could be narrowed; this potential influence needs confirmation in further studies.

4.2. Property-based derivation method

So far, it has not been feasible to use any method to create a perfect virtual subject model that precisely represents the biodynamic properties of the subjects participating in an experimental study. This is not only because any measurement could include some errors, but also because the accuracy of the biodynamic properties that can be identified from the modeling depends on the specific model structure and the techniques used to create the model. Therefore, the property-based derivation method is also an approximation method, and the response derived with this method could vary with different models. The acceptability of the derived response thus depends on the purpose of the study or the application of the model. Although the model (Fig. 2) used in this study does not provide an accurate simulation of every detailed response, the agreements between the modeling results and the laboratory-measured data are very reasonable, as shown in Fig. 5. This suggests that this model is sufficient to represent the major dynamic features of the hand–arm system along the forearm direction, and it is thus sufficient for many applications.

As demonstrated in this study, the representative model can be built using two different approaches: (a) to directly create the model using the mean BR derived from the response-based averaging method and (b) to create a model for each subject and to construct the final model based on the averages of the parameter values of the individual subject models. The model parameter values derived from these two approaches are also similar, except those for the first resonance, as presented in the last two columns of Tables 2 and 3. As shown in Fig. 6, when the number of subjects increases to a certain level, the response-based modeling results are surprisingly similar to those derived from the property-based method, except the phase angles at the low frequencies ($\leqslant 16$ Hz) related to the first resonance. These observations suggest that the models developed using the two approaches can be practically the same if a sufficient number of subjects are used to measure the biodynamic response for the construction of a representative model. Therefore, if each individual's dynamic response is not of concern, it is not necessary to use the relatively more expensive property-based approach to develop the model.

However, these observations do not mean that the property-based derivation method has little value for any application. Increasing the number of subjects in an experiment could be more expensive than building a unique model for each subject. When the response peaks occur in a wide frequency range, and it is not clear whether the response-based averaging effects are small, the use of the property-based derivation method is a reliable choice. The biodynamic response is usually measured under limited conditions. When the models for two values of a specific influencing factor (e.g., 50 N grip and 100 N grip) are developed based on the available experimental data, the response for intermediate values of that factor can be estimated using the property-based derivation method by assigning proportional weighting to each of the original models. The property-based method can also be used to take into account the nonlinear behaviors of the hand–arm system. For example, if three or more original models for an influencing factor (e.g., hand force, hand size, or arm posture) are available, a nonlinear interpolation method can be considered to estimate each of the parameters of the intermediate model. This property-based approach may also be used to establish the percentile distribution of the biodynamic response, and to derive the representative biodynamic response from the BR data reported by different laboratories or studies, especially when only a few sets of data are available. It is also appropriate to use the number of subjects that participate in each study as a weighting factor in the derivation of the representative biodynamic response.

It is noted that the modeling method used in this study can be further improved. As observed in this study, there are some uncertainties in determining several model parameters (i.e., M_0, c_0, and k_0) related to the first resonance; these model elements are relatively farther away from the physical locations where the biodynamic responses at the fingers and palm are measured. As a result, the values of the first damping ratio for some subjects are largely different, as shown in Tables 2 and 3. Such large differences may be unrealistic. Because it is very difficult to accurately measure the low frequency response [25], the biodynamic response at frequencies below 10 Hz may not be measured or the measured low frequency data could not be reliable; in such cases, the first resonant frequency and damping ratio cannot be reliably determined. Besides the large arithmetical averaging effects in the first resonant frequency region, this is also one of the reasons that there are some large differences between the first resonance parameters derived from the two modeling approaches, as presented in the last two columns of Tables 2 and 3. If some additional response information on the elbow and shoulder such as their vibration transmissibility is also measured, together with the driving-point biodynamic response, this uncertainty may be overcome. Fortunately, this uncertainty does not affect the objectives or conclusions of this study.

5. Conclusion

This study proposed a property-based method for deriving representative biodynamic responses of the hand–arm system; this property-based BR was used as a baseline to evaluate the conventional response-based derivation method. This study found that the response-based method could generate some systematic errors. The range of the subjects' natural frequencies in each resonance mode, the mode's damping ratio, and the number of subjects participating in the experiment are among the major factors influencing the level of the errors. An effective and practical approach for reducing the potential error is to increase the number of subjects in the BR measurement. On the other hand, the property-based derivation method can be generally used to obtain the representative response, but it is less efficient than the response-based derivation method.

Disclamers

The content of this publication does not necessarily reflect the views or policies of the National Institute for Occupational Safety and Health (NIOSH), nor does mention of trade names, commercial products, or organizations imply endorsement by the US Government.

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