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Unified working regime of irreversible Carnot-like heat engines with nonlinear heat transfer laws

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Abstract

We present the results of efficiency and power output for irreversible Carnot-like heat engines with nonlinear inverse, Dulong–Petit and Stefan–Boltzmann heat transfer laws when optimized with a recent criterion. A unified working regime is found intermediate between those predicted by the maximum efficiency and maximum power for all realistic values of the parameters accounting for the irreversibilities: finite rate heat transfer between the working fluid and the external heat sources, internal dissipation in the working fluid, and heat leak between reservoirs. © 2002 Elsevier Science Ltd. All rights reserved.

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1. Introduction

It has been stressed on several occasions that it would be desirable for the efficiency of a real heat engine to be larger than its maximum power efficiency and also have a power over the value predicted by the maximum efficiency regime [1]. Along this line, several thermodynamics and thermoeconomics optimization criteria have been proposed in the context of finite time thermodynamics. See Refs. [2,3] for two recent reviews on this and related subjects. In particular, the so-called ecological criterion (best compromise between power output and entropy generation) has been applied to analyze endoreversible Carnot-like heat engines with linear [4] and nonlinear Dulong–Petit [5] heat transfer laws. From these studies, it was found that in the endoreversible limit, the efficiency at maximum ecological function can be expressed *approximately* as the

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semi-sum of the Carnot efficiency and the efficiency at maximum power (semi-sum property), independently of the used heat transfer law. On the contrary, the efficiency under maximum power changes with the heat transfer law [6]. An extension of this property to heat engines with internal irreversibilities in the working fluid has also been developed [7].

In a recent paper [8], we have reported a unified optimization criterion (the Ω criterion), based on the analysis of the best compromise between energy benefits and losses, which apply for microscopic and macroscopic (traditional) energy converters. The two main features of this criterion are its independence of any environment parameter and its prediction of an optimum working regime intermediate between those obtained under maximum efficiency (or coefficient of performance) and under maximum useful energy (power output in a heat engine, cooling power in a refrigerator and heating power in a heat pump) criteria. In such work, the considered irreversible Carnot-like heat engines were assumed to have linear heat transfer laws.

The main goal of this work is to show the predictions of this optimum operating Ω criterion in Carnot-like heat engines but for the usual nonlinear heat transfer laws (inverse, radiative and Dulong–Petit laws), whose importance have been pointed out by different authors. Although the criterion is easily applicable to any cyclic model, here we choose a standard irreversible Carnot-like model because of its simplicity to account for the main irreversibilities that usually arise in real heat devices: finite rate heat transfer between the working fluid and the external heat sources, internal dissipation of the working fluid, and heat leak between reservoirs. For this cycle model, we have analyzed three working regimes: maximum efficiency, maximum work and maximum Ω. For the three nonlinear heat transfer laws considered, it is found that the efficiency under maximum Ω conditions is located between the maximum possible efficiency and the efficiency at maximum power output, while the power at maximum Ω conditions is located above the power at maximum efficiency. These results could be considered as a generalization to the broader irreversible conditions of the semi-sum property and to reinforce the Ω criterion as an optimum working regime.

2. The criterion and the model

Briefly, the Ω criterion [8] states a compromise between energy benefits and losses for a specific job and formally can be expressed as

$$\Omega(x;\{\alpha\}) = \frac{2z(x;\{\alpha\}) - z_{\min}(\{\alpha\}) - z_{\max}(\{\alpha\})}{z(x;\{\alpha\})} E_{\mathrm{u}}(x;\{\alpha\}), \tag{1}$$

where x denotes an independent variable, $\{\alpha\}$ denotes a set of parameters which can be considered as controls, $E_{\mathrm{u}}(x;\{\alpha\})$ is the useful energy delivered by the heat device along a given, nonideal process from the conversion of an input energy $E_{\mathrm{i}}(x;\{\alpha\})$, and $z(x;\{\alpha\})$ is the conventional efficiency of this heat device, defined as the ratio between the useful and input energies $z(x;\{\alpha\}) = E_{\mathrm{u}}(x;\{\alpha\})/E_{\mathrm{i}}(x;\{\alpha\})$, which can vary between a minimum $z_{\min}(\{\alpha\})$ and maximum $z_{\max}(\{\alpha\})$ values in the allowed range of values of x for given α's.

In particular, for finite time heat engines, Eq. (1) becomes [8]

$$\dot{\Omega} = (2\eta - \eta_{\max})|\dot{Q}_{\mathrm{H}}| = (2\eta - \eta_{\max})|\dot{W}|/\eta, \tag{2}$$

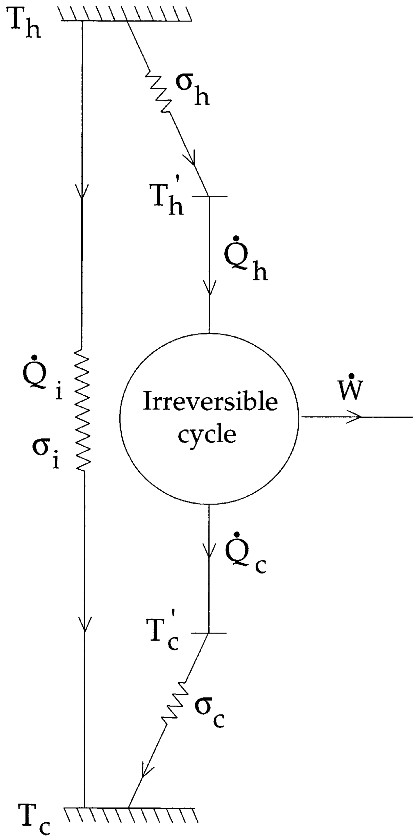

Fig. 1. Schematic diagram of the considered irreversible Carnot cycle.

where $|\dot{Q}_H|$ is the rate of heat supply (input energy), $|\dot{W}|$ is the power delivered (the useful energy), η the thermal efficiency and η_{max} the maximum possible value of η for given values of the controls.

The theoretical model we consider is the usual steady flow (continuous) irreversible Carnot power cycle sketched in Fig. 1, where $|\dot{Q}_h|$ and $|\dot{Q}_c|$ are, respectively, the rate of heat supplied by the hot reservoir at temperature T_h and absorbed by the heat sink at temperature T_c, $|\dot{Q}_i|$ is the rate of heat leak between the external sources, $|\dot{W}|$ is the power output per cycle, $T_h'(< T_h)$ and $T_c'(> T_c)$ are, respectively, the temperatures of the working fluid along the upper and the lower isothermal processes, σ_h and σ_c are, respectively, the *external* hot end (steam boiler) and cold end (condenser) thermal conductances, and σ_i is the *internal* heat conductance.

3. Inverse heat transfer law

The importance of the inverse heat transfer law lies in its connection with the phenomenological law of irreversible thermodynamics. It has been analyzed with similar irreversible models under the maximum power regime and the relation between power and efficiency (the so-called fundamental optimal relation) was obtained [9,10]. Here, we shall focus on the results concerning the $\dot{\Omega}$ criterion. The involved heat flows are given by (see Fig. 1)

$$|\dot{Q}_{\rm h}| = \sigma_{\rm h}\left(\frac{1}{T'_{\rm h}} - \frac{1}{T_{\rm h}}\right) = \frac{\sigma_{\rm h}}{T_{\rm h}}(a_{\rm h} - 1), \tag{3}$$

$$|\dot{Q}_{\rm c}| = \sigma_{\rm c}\left(\frac{1}{T_{\rm c}} - \frac{1}{T'_{\rm c}}\right) = \frac{\sigma_{\rm h}}{T_{\rm h}}\frac{1}{\tau\sigma_{\rm hc}}\left(1 - \frac{1}{a_{\rm c}}\right), \tag{4}$$

$$|\dot{Q}_{\rm i}| = \sigma_{\rm i}\left(\frac{1}{T_{\rm c}} - \frac{1}{T_{\rm h}}\right) = \frac{\sigma_{\rm h}}{T_{\rm h}}\sigma_{\rm ih}\left(\frac{1}{\tau} - 1\right), \tag{5}$$

where $a_{\rm h} = T_{\rm h}/T'_{\rm h} \geqslant 1$, $a_{\rm c} = T'_{\rm c}/T_{\rm c} \geqslant 1$, $\tau = T_{\rm c}/T_{\rm h} \leqslant 1$, $\sigma_{\rm hc} = \sigma_{\rm h}/\sigma_{\rm c}$, and $\sigma_{\rm ih} = \sigma_{\rm i}/\sigma_{\rm h}$. The internal irreversibilities, accounted by a global parameter I, and the working fluid temperatures are related through the Clausius inequality as [9,10]

$$\frac{|\dot{Q}_{\rm h}|}{T'_{\rm h}} = I\frac{|\dot{Q}_{\rm c}|}{T'_{\rm c}} \quad (0 < I \leqslant 1). \tag{6}$$

At sight of Eqs. (3), (4) and (6), $a_{\rm c}$ can be expressed in terms of $a_{\rm h}$, τ, $\sigma_{\rm hc}$ and I. Then, we will consider $a_{\rm h}$ as our independent variable, while τ, $\sigma_{\rm hc}$, $\sigma_{\rm ih}$ and I will be considered as the set $\{\alpha\}$ of controls. In particular, for this heat transfer law, the Clausius inequality gives $a_{\rm c} = (I - \sqrt{I^2 - 4\tau^2 a_{\rm h}(a_{\rm h} - 1)I\sigma_{\rm hc}})/2\tau^2 a_{\rm h}(a_{\rm h} - 1)I\sigma_{\rm hc}$. Since $a_{\rm c}$ should be positive, then $a_{\rm h}(a_{\rm h} - 1) \leqslant I/(4\tau^2\sigma_{\rm hc})$. This inequality is a constraint among $a_{\rm h}$ and the involved irreversibility parameters, which should be taken into account in any calculation. All results we show below have been obtained from a Mathematica code.

Power $|\dot{W}|(a_{\rm h}, \tau, I, \sigma_{\rm hc}) = |\dot{Q}_{\rm h}| - |\dot{Q}_{\rm c}|$, efficiency $\eta(a_{\rm h}, \tau, I, \sigma_{\rm hc}, \sigma_{\rm ih}) = |\dot{W}|/(|\dot{Q}_{\rm h}| + |\dot{Q}_{\rm i}|)$, and $\dot{\Omega}(a_{\rm h}, \tau, I, \sigma_{\rm hc}, \sigma_{\rm ih}) = (2\eta - \eta_{\rm max})|\dot{W}|/\eta$ are plotted in Fig. 2 in terms of $a_{\rm h}$ for a realistic set of values of the controls. The results for the power and $\dot{\Omega}$ are normalized to $\sigma_{\rm h}/T_{\rm h}$. Note in this figure how the typical parabolic behavior of the efficiency and power with respect to $a_{\rm h}$ also applies for the $\dot{\Omega}$ function. More important is the fact that the efficiency under maximum Ω conditions, $\eta_{\max\dot{\Omega}}$, is located between the maximum efficiency, $\eta_{\rm max}$, and the efficiency at maximum power, $\eta_{\max|\dot{W}|}$. The values of $a_{\rm h}$ for which these optimal efficiency values apply, satisfy the inequality

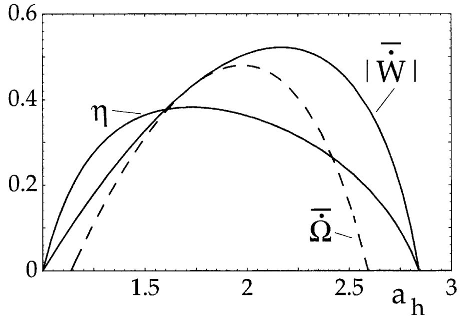

Fig. 2. Efficiency η, dimensionless power $|\bar{\dot{W}}|$ and dimensionless $\bar{\dot{\Omega}}$ in terms of $a_{\rm h}$ for the inverse heat transfer law and the following values of the controls: $\tau = 0.2$, $I = 0.9$, $\sigma_{\rm hc} = 1$, $\sigma_{\rm ih} = 0.1$.

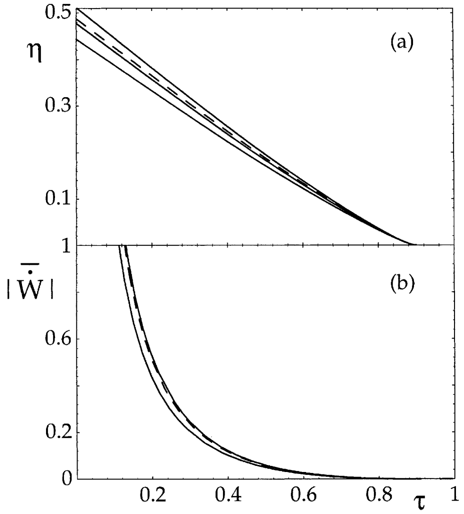

Fig. 3. Efficiency and power for the inverse heat transfer law with $I = 0.9$, $\sigma_{hc} = 1$, $\sigma_{ih} = 0.1$. (a) Maximum efficiency $\eta_{\max}(\tau)$ (upper solid line), efficiency at maximum $\dot{\Omega}$ conditions $\eta_{\max\dot{\Omega}}(\tau)$ (intermediate dashed line), efficiency at maximum power $\eta_{\max|\dot{W}|}(\tau)$ (lower solid line). The intermediate solid line is $[\eta_{\max}(\tau) + \eta_{\max|\dot{W}|}(\tau)]/2$. (b) Maximum power $|\dot{W}|_{\max}$ (upper solid line), power at maximum $\dot{\Omega}$ conditions $|\dot{W}|_{\max\dot{\Omega}}$ (intermediate dashed line) and power at maximum efficiency $|\dot{W}|_{\max\eta}$ (lower solid line).

$a_{\mathrm{h},\max|\dot{W}|} > a_{\mathrm{h},\max\Omega} > a_{\mathrm{h},\max\eta}$. Since T_h is fixed, the above inequality means that the maximum efficiency regime needs greater T'_h values than the $\dot{\Omega}$ regime which, in turn, needs greater T'_h values than the maximum efficiency regime. The τ-behavior of the three optimized efficiencies is plotted in Fig. 3a and for the three optimized powers in Fig. 3b. These figures illustrate clearly how the $\dot{\Omega}$ regime is intermediate between the maximum efficiency and maximum power regimes. In particular, we note in Fig. 3a how $\eta_{\max\dot{\Omega}}$ is practically coincident with the semi-sum of the maximum efficiency and efficiency at maximum power and in Fig. 3b how the power for the $\dot{\Omega}$ regime is very close to the maximum power. Thus, the $\dot{\Omega}$ criterion gives a working regime for which the efficiency is greater than the maximum power efficiency and the power is close to its maximum possible value. The above features also apply for all sets of values of the controls we have checked.

4. Dulong–Petit and Stefan–Boltzmann heat transfer laws

The Dulong–Petit heat transfer law $(\Delta T)^n$ with $n = 5/4$ has been considered as a phenomenological fit to take into account combined conductive–radiative heat transfers between the working fluid and the external heat reservoirs [5,7,9]. The Stefan–Boltzmann law, $T^k - T'^k$, with $k = 4$, applies for radiative heat engines used in solar energy conversion [11]. As before, we restrict ourselves to the results concerning the Ω criterion. The involved heats are given by

$$|\dot{Q}_\mathrm{h}| = \sigma_\mathrm{h}(T_\mathrm{h}^k - T_\mathrm{h}'^k)^n = \sigma_\mathrm{h} T_\mathrm{h}^{(kn)}\left(1 - \frac{1}{a_\mathrm{h}^k}\right)^n, \tag{7}$$

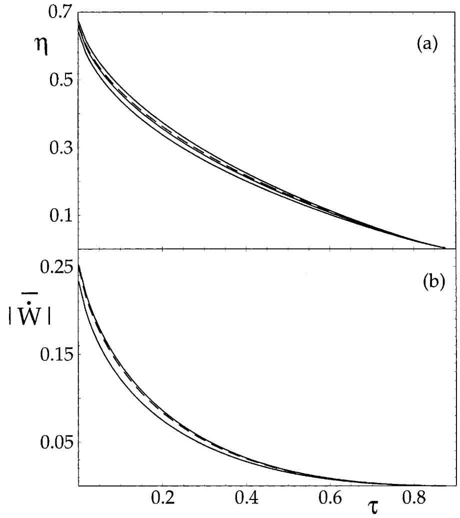

Fig. 4. As Fig. 3a and b but for the Dulong–Petit heat transfer law.

$$|\dot{Q}_\mathrm{c}| = \sigma_\mathrm{c}(T_\mathrm{c}^{\prime k} - T_\mathrm{c}^k)^n = \sigma_\mathrm{h} T_\mathrm{h}^{(kn)} \frac{\tau^{(kn)}}{\sigma_\mathrm{hc}}(a_\mathrm{c}^k - 1)^n, \tag{8}$$

$$|\dot{Q}_\mathrm{i}| = \sigma_\mathrm{i}(T_\mathrm{h}^k - T_\mathrm{c}^k)^n = \sigma_\mathrm{h} T_\mathrm{h}^{(kn)} \sigma_\mathrm{ih}(1 - \tau^k)^n, \tag{9}$$

with $k = 1$, $n = 5/4$ for the Dulong–Petit and $k = 4$, $n = 1$ for the Stefan–Boltzmann laws. The implicit constraints between the independent variable and the controls $a_\mathrm{c} = a_\mathrm{c}(a_\mathrm{h}, \tau, I, \sigma_\mathrm{hc})$, coming from the Clausius inequality are now given by $\sigma_\mathrm{hc} a_\mathrm{h}^{1-n}(a_\mathrm{h} - 1)^n = I\tau^{n-1}(a_\mathrm{c} - 1)^n / a_\mathrm{c}$ (with $n = 5/4$) for the Dulong–Petit and by $\sigma_\mathrm{hc}(a_\mathrm{h} - a_\mathrm{h}^{-3}) = I\tau^3(a_\mathrm{c}^3 - a_\mathrm{c}^{-1})$ for the Stefan–Boltzmann laws.

From Eqs. (7)–(9), it is easy to obtain the power output, the efficiency and $\dot{\Omega}$. Similar qualitative behaviors to those shown in Fig. 2 for the inverse law, are obtained with these two heat transfer laws, and they are not plotted here. The τ behavior of the optimized efficiencies and powers is more interesting. Fig. 4a shows η_max, $\eta_{\mathrm{max}\,|\dot{W}|}$ and $\eta_{\mathrm{max}\,\dot{\Omega}}$, while Fig. 4b shows $|\dot{W}|_\mathrm{max}$, $|\dot{W}|_{\mathrm{max}\,\dot{\Omega}}$, and $|\dot{W}|_{\mathrm{max}\,\eta}$ (in reduced units $\sigma_\mathrm{h} T_\mathrm{h}^{5/4}$) for the Dulong–Petit heat law. Fig. 5a and b show the corresponding values for the Stefan–Boltzmann law (the powers in reduced units $\sigma_\mathrm{h} T_\mathrm{h}^4$). In spite of the different qualitative behavior of these functions, depending on the particular heat transfer law, we stress from Figs. 4 and 5 the intermediate character of the efficiency and power when optimized under the $\dot{\Omega}$ criterion in relation to the maximum efficiency and maximum power regimes. As before, we have checked that these features also apply for all sets of realistic values of the controls.

5. Endoreversible limits

When $I = 1$ and $\sigma_\mathrm{i} = 0$, the overall irreversibilities are those coming from the coupling between the working fluid and the external reservoirs, i.e. the heat engine is endoreversible. In this case, $\eta_\mathrm{max} = 1 - \tau \equiv \eta_\mathrm{C}$, independently of the relative conductance σ_hc. On the other side, the entropy

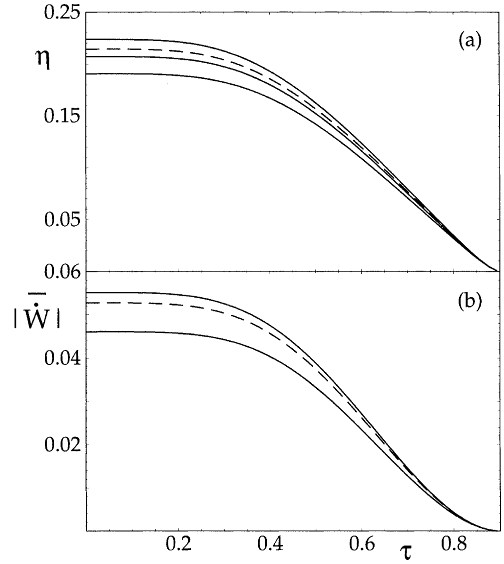

Fig. 5. As Fig. 3a and b but for the Stefan–Boltzmann heat transfer law.

generation is given by $\dot{S} = (|\dot{Q}_c|/T_c) - (|\dot{Q}_h|/T_h) = (|\dot{Q}_h|/T_c)(\eta_C - \eta)$. Thus, the ecological function (as defined by Angulo [4]) $E = |\dot{W}| - T_c\dot{S} = [\eta - (\eta_C - \eta)]|\dot{Q}_h| \equiv \dot{\Omega}$, i.e. the ecological and the Ω criteria are identical in the endoreversible limit. However, they may differ under irreversible conditions [8]. In this endoreversible limit and for the three nonlinear heat transfer laws under consideration, the efficiency under maximum power conditions is σ_{hc}-dependent [12]. Indeed, the same happens for the efficiency under maximum $\dot{\Omega}$ conditions. Although the results are not plotted, in this limit and for all realistic values of σ_{hc}, the close coincidence between $\eta_{\max \dot{\Omega}}$ and the semi-sum property for the three nonlinear heat transfer laws under consideration has been found.

6. Conclusions

In summary, we have found that both irreversible and endoreversible Carnot-like heat engines with inverse, Dulong–Petit and Stefan–Boltzmann heat transfer laws and working under optimal Ω conditions have: (i) an efficiency lying between the maximum possible efficiency and the efficiency at maximum power and (ii) a power close to the maximum possible value. These findings, together with those obtained in Ref. [8] for the linear heat transfer law, can be considered a generalization to irreversible situations of the previous results concerning the semi-sum property.

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