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Calculation of the importance-weighted neutron generation time using MCNIC method

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Abstract

In advanced nuclear power systems, such as ADS, the need for reliable kinetics parameters is of considerable importance because of the lower value for β_{eff} due to the large amount of transuranic elements loaded in the core of those systems. All reactor kinetic parameters are weighted quantities. In other words each neutron with a given position and energy is weighted with its importance. Neutron generation time as an important kinetic parameter, in all nuclear power systems has a significant role in the analysis of fast transients. The difference between non-weighted neutron generation time; Λ; standard in most Monte Carlo codes; and the weighted one $\Lambda^{\dagger}$ can be quite significant depending on the type of the system. In previous work, based on the physical concept of neutron importance, a new method; MCNIC; using the MCNP code has been introduced for the calculation of neutron importance in fissionable assemblies for all criticality states. In the present work the applicability of MCNIC method has been extended for the calculation of the importance-weighted neutron generation time. The influence of reflector thickness on importance-weighted neutron generation time has been investigated by the development of an auxiliary code, IWLA, for a hypothetic assembly. The results of these calculations were compared with the non-weighted neutron generation times calculated using the Monte Carlo code MCNP. The difference between the importance-weighted and non-weighted quantity is more significant in a reflected system and increases with reflector thickness.

1. Introduction

The issue of determining the accurate value of kinetic parameters such as the neutron generation time has been demonstrated to be of major importance in designing reactor safety features. In advanced nuclear power systems, such as ADS, the need for reliable kinetics parameters is of considerable importance because of the lower value for β_{eff} due to the large amount of transuranic elements loaded in the core of those systems. The calculation of these parameters is usually performed with deterministic codes. There are three general available methods in this regard (Verboomen et al., 2006): (i) the direct integration method (Keepin, 1965); (ii) the perturbation method (Henry, 1958); and (iii) the iterated fission probability method (Bohl and Margolis, 1964).

For advanced nuclear power systems such as ADS, the Monte Carlo method is the preferred calculation tool since it has the ability to handle nuclear data not only in its most basic but also most complex form: continuous energy cross-sections, complex interaction laws, detailed energy-angle correlations, multi-particle physics, S(α, β) tables for thermal neutron scattering by molecules and crystalline solids, unresolved resonance probability tables. It also can handle very complex 3D geometries. As a result, normal critical systems as well as sub-critical systems with an external source can all be calculated with a single code, practically without making any approximation. Due to this growing use of Monte Carlo codes one now also desires the calculation of the kinetic parameters by such codes.

Among various time interval definitions characterizing the life of a neutron through a given system, the neutron

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generation time has special significance in conjunction with reactor point kinetics.

Since this neutron time parameter has a great importance when analyzing dynamical reactor behavior, it is of interest to find it by simulations based on deterministic and Monte Carlo transport expressions that can be used to determine this parameter.

Among reliable statistical and deterministic codes utilized in reactor neutronic calculations, MCNP (Briesmeister, 2000) is one of the most frequently used Monte Carlo neutron transport codes which are preferred for the neutronic calculations of advanced nuclear power systems. Despite its considerable advantages in solving problems with complex geometries, MCNP fails in predicting the accurate value of neutron generation time (i.e. importance-weighted) especially in highly reflected reactors. This drawback originates from the fact that all quantities simulated in this code are non-importance-weighted.

All parameters in reactor kinetics theory are weighted quantities (Henry, 1975). In other words, each neutron with a given position and energy is weighted with its importance, which is the sum of its contributions to the system multiplication factor, for all generations starting from that particular neutron (Feghhi et al., 2007).

The difference between the non-weighted neutron generation time and the importance-weighted neutron generation time as an important kinetic parameter can be quite significant depending on the type of the system. This difference is very substantial especially in highly reflected reactors (Verboomen et al., 2006).

The problem of calculating the importance-weighted point kinetics parameters was investigated by many authors, which will be dealt with further in detail. But it is still an open question, how the weight function should be calculated for nuclear power systems with complex geometries especially in a source-driven case. The aim of the present work is also to give an answer to this question through calculating the importance-weighted neutron generation time.

In previous work (Feghhi et al., 2007), we proposed a method based on the physical concept of neutron importance; MCNIC; to calculate the neutron importance function in fissionable assemblies of complex geometries for all criticality states, using the MCNP Code Monte Carlo calculations.

It is the objective of this study to introduce a new approach based on the MCNIC method for the calculation of the importance-weighted neutron generation time in fissionable assemblies of complex geometries regardless of the state of criticality.

This new approach is based on a deterministic expression of importance-weighted neutron generation time derived using the neutron-balance model proposed by Weinberg and Wigner (1958). The MCNP code has been utilized in applying the MCNIC method; for calculating multi-group importance function; and calculating multi-group forward flux. For this propose an auxiliary computer program, IWLA, has been developed.

2. Kinetics equations and the requiring weight function

The conventional kinetics equations were derived by Henry (1975) starting from the time dependent neutron transport equation and their limitations and capabilities for critical reactor analysis have been investigated in great detail (Gyftopoulos, 1964; Ott and Meneley, 1969; Yasinsky and Henry, 1965).

The purpose behind the formulation of the kinetics equations is to derive a lumped model that describes the change in the average level of the flux, therefore, the neutron flux is factorized in amplitude and shape functions. In flux factorization the neutron spatial and energy distributions, in contrast to separation of variables may still depend on time. However, an imposed constraint condition which is necessary to define precisely the amplitude and shape functions introduces a weight function that allows manipulation of the kinetics equations in a way that simplifying assumptions (such as the point kinetics approximation) can be applied more effectively.

As long as the actual time-dependent flux shape is calculated, the weight function can be any function that is defined over the same energy and spatial domain as the flux. Using this weight function after some manipulations, one can arrive at the conventional point kinetics equations, which solution will be exactly the same as the solution of time-dependent continuous energy neutron transport equation for $\Phi(\vec{r}, E, t)$ (Henry, 1975). The error is introduced when we modify the equations to better cope with an approximate representation of the time-dependent flux shape, e.g., the point kinetics approximation. In that case, the weight function becomes useful because it leaves us with the possibility of freely choosing it in a manner to better suit a point kinetics approximation. Within a perturbation theory approach it is shown for a critical reactor, that adjoint flux weighting eliminates the influence of first-order flux shape changes on the reactivity, and therefore, also reduces the error in the approximation of amplitude function (Henry, 1975). The neutron balance equation for a reactor with an independent source, such as ADS, is mathematically an inhomogeneous problem. In strict terms, separation of variables is not possible for such cases. Thus, the point kinetics approximation becomes questionable.

The procedure for generating a weight function for nuclear power systems as well as source driven ones in a deterministic fashion have been investigated by many authors.

For near critical systems, it is well known (Dorning and Spiga, 1978; Ott and Neuhold, 1985, p. 57) that the best weighting function is the initial critical adjoint flux and the shape function is simply the initial critical flux. Using the adjoint flux as weighting function has the benefit of producing adjoint weighted kinetics parameters. Since a source-free adjoint weighting function does not correspond to the actual state of a source-driven system the physical meaning of the point kinetics parameters is not clear in this case. Furthermore, the required weighting function for near critical systems used in the calculation of the adjoint-weighted neutron generation time is usually determined by calculating adjoint flux while solving the adjoint-weighted transport equation based on deterministic methods. But, in complex geometries these calculations are usually very difficult.

In what follows, through calculating the importance-weighted neutron generation time as an important kinetic parameter, we will proceed to introduce a new approach in calculating the importance-weighted kinetics parameters based on the weight function calculated by MCNIC method.

3. Importance-weighted and non-importance-weighted neutron generation time

The phrase "neutron lifetime" has been used too often in the past to describe a wide variety of time intervals associated with a neutrons track through a given system.

In an effort to clarify the meaning of some of the more important time intervals associated with the life of a neutron, Spriggs et al. (1997) suggested that these various time intervals be divided into three categories: (1) neutron lifespans, (2) reaction rate lifetimes, and (3) neutron generation times. They also defined these three time intervals and gave deterministic and Monte Carlo transport expressions that can be used to calculate them.

A neutron generation time is a special case of reaction rate lifetime; it is the mean time per neutron (or unit of importance) between fission production events.

The use of the non-importance-weighted neutron generation time Λ; standard in most Monte Carlo codes; as an approximation of the importance-weighted one $\Lambda^{\dagger}$ can lead to a serious disadvantage.

Ussachoff (1955) derived the point kinetic equations from the Boltzmann transport equation. From that derivation, he obtained an expression for the neutron generation time, $\Lambda^{\dagger}$, as a function of the angular fluxes and angular adjoint fluxes, It has the general form of

$$\Lambda^{\dagger} = \frac{1}{F}\int\int\int \frac{1}{\upsilon(E)}\Phi^{\dagger}(\vec{\mathbf{r}},\hat{\Omega},E)\Phi(\vec{\mathbf{r}},\hat{\Omega},E)\mathrm{d}\vec{r}\,\mathrm{d}\hat{\Omega}\,\mathrm{d}E,$$
$$F = \int \mathrm{d}\hat{\Omega}\int \mathrm{d}E\int\int\int \chi(E)\gamma\Sigma_f(\vec{\mathbf{r}},E') \times \Phi^{\dagger}(\vec{\mathbf{r}},\hat{\Omega},E)\Phi(\vec{\mathbf{r}},\hat{\Omega}',E')\mathrm{d}\vec{r}\,\mathrm{d}\hat{\Omega}'\,\mathrm{d}E' \quad (1)$$

as an importance-weighted quantity, where all the symbols follow the same meaning as in Bell and Glasstone (1976). Here $\Lambda^{\dagger}$ is defined as the importance-weighted neutron generation time and is equal to $\frac{\tau_r^{\dagger}}{k_{\mathrm{eff}}}$ [1]. This generation time represents the mean time per unit neutron importance between the productions of fission neutrons; it does not include any contributions from (n,xn) reactions, (γ, n) reactions, etc., or intrinsic/external neutron sources. From its definition, we also note that the importance-weighted neutron generation time is only equal to the importance-weighted removal lifetime at delayed critical.

In MCNP4B and later versions, two different types of lifetime parameters are treated: lifespans and lifetimes. The neutron mean generation time; Λ; is neither a lifetime nor a lifespan and is not represented in MCNP.

MCNP calculates the non-importance-weighted prompt removal lifetime, τ_r, that can be significantly different in a multiplying system (Briesmeister, 2000). Therefore, the non-weighted neutron generation time, Λ, can be determined as $\frac{\tau_r}{k_{\mathrm{eff}}}$.

4. Materials and methods

In what follows, we will focus on the calculation of $\Lambda^{\dagger}$ with a new approach based on the weight function obtained from MCNIC method. This method is easy to implement because, it does not require any changes in the MCNP Monte Carlo code. The methodology describe here has been applied to MCNP4C for two hypothetic bare and graphite reflected systems in calculation of their importance-weighted neutron generation time.

4.1. The MCNIC method

Suppose, $k_{\mathrm{s}}(\vec{r},E)$ is the multiplication factor of an isotropic neutron source with energy E located at $\vec{r}$ in a reactor. For such a source, neutron importance function is determined by

$$\Phi^{\dagger} = \sum_{i=1}^{\infty} k_{\mathrm{s}}^{i} = k_1 + k_1k_2 + \cdots + k_1k_2\cdots k_i + \cdots \quad (2)$$

where k_i is the value of system multiplication factor for the ith generation of neutrons. Thus, the importance of a neutron with a given position and energy is the sum of its contributions to the system multiplication factor, for all generations starting from that particular neutron. The procedure of MCNIC method for calculation of neutron importance function at position $\vec{r}_0$ and energy E_0; $\Phi^{\dagger}(\vec{r}_0,E_0)$; is as follow:

An isotropic neutron source with energy E_0 is defined at $\vec{r}_0$ and the values of k_{i} are calculated in an analogue forward MCNP simulation of the problem. Then $\Phi^{\dagger}(\vec{r}_0,E_0)$ can be calculated using Eq. (2).

Since the effective multiplication factor of a sub-critical system is always less than unity, k_{i} becomes convergent to $k_{\mathrm{eff}} < 1$, regardless of the energy and location of neutron source and their importance function will have a finite value. Therefore

$$\Phi^{\dagger} = \sum_{i=1}^{\infty} k_{\mathrm{s}}^{i} = \frac{k_{\mathrm{s}}}{1-k_{\mathrm{s}}} \quad (3)$$

[1] Importance-weighted prompt removal lifetime.

The given procedure leads to a meaningful finite importance function, only for sub-critical case; $k_{\text{eff}} < 1$. In the other words, if $k_{\text{eff}} \geqslant 1$, then the Eq. (2) will not be convergent. In order to generalize the method for all criticality conditions we introduce a "relative importance" function.

After some transient cycles, depending on the neutron mean free path in the system, in order to give enough time for the fission source to be established, the multiplication factor of the subsequent generations converges to k_{eff}, regardless of criticality condition.

$$\lim_{i \to \infty} k_i = k_{\text{eff}} \tag{4}$$

If the number of transient cycles is equal to n, then the neutron population; per source particle; after n cycles is:

$$N_T = (k_1 + k_1k_2 + k_1k_2k_3 + \cdots + k_1k_2\cdots k_n) \tag{5}$$

Considering T, as the time taken to reach transient cycles, the neutron population after T, will be,

$$N(t) = N_T + N_{\text{n}} \sum_{i=1}^{\frac{(t-T)}{\ell}} k_{\text{eff}}^i, \ (t \geqslant T + \ell) \tag{6}$$

where ℓ is the mean neutron lifetime in the system and N_n is the neutron population generated in nth cycle.

$$N_n = (k_1k_2k_3\cdots k_n) \tag{7}$$

According to Eq. (6) in sub-critical condition, namely $k_{\text{eff}} < 1$, the final value of $N(t)$ will be finite and equal to the neutron importance obtained from Eq. (3) and generally $\Phi^\dagger$ can be defined as:

$$\Phi^\dagger = \lim_{t \to \infty} N(t) \tag{8}$$

If $k_{\text{eff}} > 1$, then the neutron population generated in the system asymptotically will increase with the rate of $(k_{\text{eff}})^{\frac{(t-T)}{\ell}}$ when $t \to \infty$. Therefore, the definition of neutron importance can be changed from an absolute quantity to a relative quantity without affecting the validity of the method. Therefore, the ratio of neutrons importance with different energies and places is equal to the ratio of total neutron population generated by them in the system when $t \to \infty$. According to Eqs. (6) and (8) it can be seen that for $k_{\text{eff}} > 1$ we have

$$\frac{\Phi_1^\dagger}{\Phi_2^\dagger} = \frac{(N_n)_1}{(N_n)_2} \tag{9}$$

where, $\Phi_1^\dagger$ and $\Phi_2^\dagger$ are neutron importance functions at $(\vec{r}_1, E_1)$ and $(\vec{r}_2, E_2)$, respectively.

Thus Eq. (6) along with Eq. (8) gives a general method for determining neutron importance in a fissionable system for sub-critical, critical and super-critical conditions.

4.2. The calculation procedure of importance-weighted neutron generation time

According to Eq. (1), importance-weighted neutron generation time can be calculated from:

$$\Lambda^\dagger = \frac{\int \mathrm{d}\vec{r} \int \mathrm{d}E \Phi^\dagger(\vec{r}, E) \frac{1}{v(E)} \Phi(\vec{r}, E)}{\int \mathrm{d}\vec{r} \int \mathrm{d}E \Phi^\dagger(\vec{r}, E) \chi_{\text{p}}(E) \int_0^\infty \bar{v}_t \Sigma_f(\vec{r}, E) \Phi(\vec{r}, E') \mathrm{d}E'} \tag{10}$$

where $\chi_{\text{p}}(E)$ is the prompt fission neutrons spectrum and

$$\left[\chi_{\text{p}}(E) \int_0^\infty \bar{v}_t \Sigma_f(\vec{r}, E') \Phi(\vec{r}, E') \mathrm{d}E'\right] \mathrm{d}E\mathrm{d}V$$

is the prompt fission neutron production rate in $dE\,dV$. In the case of fast transients delayed neutrons can be ignored. It will be assumed that scattering is isotropic, that all fission neutrons are emitted isotropically and that the cross sections are independent of the direction of travel of the incident neutron. After spatial and energy discritization, Eq. (10) can be rewritten in the form

$$\Lambda^\dagger = \frac{\sum_{m=1}^M \sum_{g=1}^G \Phi_{g,m}^\dagger (\frac{1}{v})_g \Phi_{g,m} \Delta V_m}{\sum_{m=1}^M \sum_{g=1}^G \Phi_{g,m}^\dagger \chi_{\text{g}} \sum_{g'=1}^G \bar{v}_t \sum_{f_{g',m}} \Phi_{g',m} \Delta V_m} \tag{11}$$

where M is the number of spatial meshes intervals, G is the number of energy groups, and

$$\Phi_{g,m} \equiv \frac{1}{v_m} \int_{V_m} \int_g \Phi(r, E)\, dE\, dV$$

As an overall convention, m and g subscripts represent the mesh interval and energy group averaged parameters, respectively. For the calculation of $\Lambda^\dagger$ through Eq. (11), $\Phi^\dagger$ and Φ are determined by MCNIC method and MCNP, respectively.

To facilitate the calculation of $\Lambda^\dagger$, we also developed an auxiliary Fortran code called IWLA which wraps itself around any version of MCNP to automate the calculation of $\Lambda^\dagger$. The input for IWLA is simply an MCNP input file along with a set of values for the number of spatial meshes intervals, the number of energy groups as well as its structure and proportionally the values for χ_{g}, $\bar{v}_t \Sigma_{fg}$ and $(\frac{1}{v})_g$. The code then prepares separate input files for every predetermined neutron source energy and position and runs them using the MCNP executable indicated by the user. After running each input file, IWLA reads the k_i values of that run from the output file and calculates $\Phi_{g,m}^\dagger$ So at the end we have a $G \times M$ matrix representing the neutron importance function. Using these, along with matrix of the very dimensions representing the neutron flux from an individual forward run, one can calculates importance-weighted neutron generation time in the desired system.

5. Results and discussion

To illustrate this new approach in calculation of the importance-weighted neutron generation time and also to highlight the influence of the reflector thickness, in highly reflected reactors, two hypothetic spherical bare and reflected reactors has been considered. For the sake of more clarification, graphite as a good reflector material that has a high scattering to absorption cross section ratio has been chosen. A sphere of ^{235}U with 6 cm radius was considered for bare system, the radius was increased up to 12 cm and for reflected one, graphite reflector with different thicknesses up to 18 cm was considered.

The importance-weighted neutron generation time was evaluated for bare and reflected systems, then compared to the non-importance weighted neutron generation time ascertained from MCNP simulation. The results are shown in Figs. 1 and 2.

In our illustrative example the difference in spatial dependency of importance-weighted neutron flux, $\Phi\Phi^\dagger$, with respect to the non-weighted flux, Φ, is the main source of difference between $\Lambda^\dagger$ and Λ. In highly reflected systems we have a considerable number of neutrons in the reflector which have much lower importance with respect to the neutrons in core region but having greater lifetimes due to multiple scattering. All of these neutrons are considered with the same weight in estimation of the total systems life time in Monte Carlo method.

Therefore, as shown in Figs. 1 and 2, the neutron generation time in reflected system, calculated by MCNP dramatically over estimates the accurate neutron generation time as obtained from the new approach.

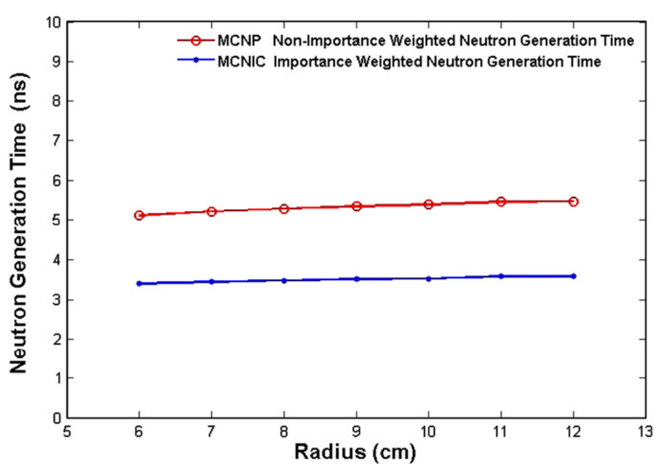

Fig. 1. Comparison of the importance-weighted and non-importance weighted neutron generation time in a hypothetical bare reactor.

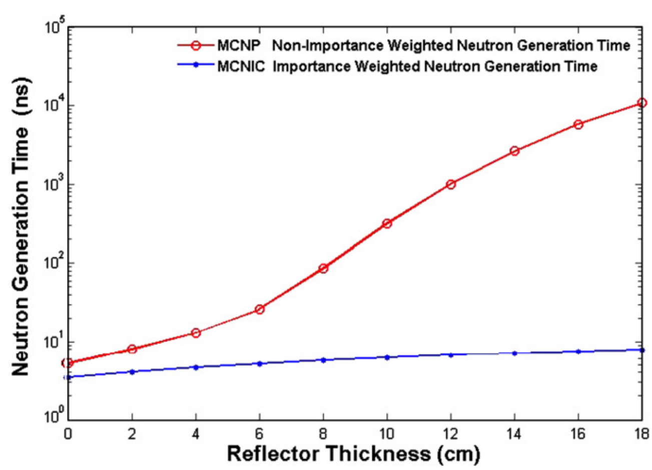

Fig. 2. Influence of the reflector thickness on importance-weighted and non-importance weighted neutron generation time in a hypothetical graphite reflected reactor.

6. Conclusion

- In conclusion, MCNP fails in predicting the accurate value of neutron generation time, especially in highly reflected reactors. This drawback originates from the fact that all quantities simulated in this code are non-importance-weighted.
- The difference between importance-weighted and non-importance weighted neutron generation time in highly reflected reactors is significantly larger than the difference between the same parameters in bare reactors. This difference is particularly important for fast fission systems with large reflector.
- We have introduced, for the first time, a new approach based on MCNIC method for the calculation of the importance-weighted neutron generation time in fissionable assemblies that can be applied to practical problems in calculation of the importance-weighted kinetic parameters (e.g. neutron generation time) in nuclear power systems of complex geometry, regardless of the criticality state.
- It should be noted that this new approach is also applicable in kinetic studies of accelerator driven subcritical reactors. In other words the MCNIC method can also include the external neutron source importance.

Our forthcoming papers will be devoted entirely to the calculation of the neutron importance function in source-driven systems and other point kinetics parameters using MCNIC method.

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