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AN APPRECIATION OF THE CONTRIBUTIONS OF NOEL CORNGOLD TO NUCLEAR REACTOR THEORY

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Abstract: Noel Corngold's 70th birthday gives us the opportunity to assess his contributions to nuclear science from his first paper in 1956 up to the present time. It would be presumptuous for me to attempt such a wide ranging review, but certainly I feel confident in discussing Corngold's work on reactor theory spanning the period 1956 - 1975. The purpose of this paper is therefore twofold: firstly to review the papers on resonance absorption, lattice physics and neutron thermalisation and secondly, to show how this work has influenced our thinking on these subjects, indeed has set a gold standard for reactor theory research which has rarely been bettered. Some remarks are also made about the work of the graduate students who have benefited from Corngold's particular style of supervision.

1. PROLOGUE AND THERMAL NEUTRONS

It is fortunate that I am an inveterate hoarder of letters and papers. My records go back to the beginning of my post-graduate career in 1958 and in some instances a lot earlier. As some have found to their cost, I can usually produce a document to prove that what Professor X said in 1962 is quite the opposite of what he said in 1972. I do not make this claim to suggest that hoarding is a virtue, indeed my wife has quite the reverse opinion. However, it does have advantages and one of them is that I have a complete set of correspondence between me and Noel Corngold starting from 1960. It was in that year that I first became aware of the name 'Noel Corngold' (to be precise, N.R.D. Corngold, three initials are so distinguished), in connection with my research work on neutron thermalisation. It was his paper entitled 'Thermalization of neutrons in infinite homogeneous systems' (Annals of Physics (NY) **6** (1959) 368-398) which caught my eye. It happened in this way. As part of my introduction to the thermalisation problem, I decided that I really ought to know more about the basic thermal neutron scattering laws and their methods of derivation. They were all based upon Fermi's pseudo-potential which he published in 1936, but the route from the pseudo-potential to $\sigma(E' \to E;\Omega' \to \Omega)$ involved what to me at the time was black magic and hand-waving. There were heavy gas approximations, a model for rigidly bound protons at zero temperature, clever tricks by Schwinger and Teller on ortho and para hydrogen and effective mass methods of Sachs and Teller. Special methods existed for crystals, others for liquids and yet more for gases. I was confused. Things looked up a bit when I came across a report written by H.L. MacMurray from Idaho Falls who put things in a far more readable form for a graduate student. Years later I met Dr. MacMurray, quite by chance, at Denver airport and was able to thank him for that report. Life got even better when I encountered George Placzek's inverse mass expansion, and all was light when, finally, I got to grips with Wick's short collision time approximation (G.C. Wick, Phys. Rev. **94** (1954) 1228).

Armed with all this background information, I then realised that I still had to solve the Boltzmann equation. Now in the short collision time approximation, there is a part of the procedure where one is faced with the following expression for the total cross section σ(E),

$$\sigma(E) = \frac{1}{2\pi m k_0} \int d\mathbf{k} \int_{-\infty}^{\infty} dt e^{it\Delta E} \langle G_{\beta\beta} \rangle$$

I will not define the symbols, readers of this paper will know what they mean. Wick then does some very cunning things by re-grouping some terms, expanding in powers of t and using the definition of the generalised function, the nth derivative of the Dirac delta function. This is used to obtain an expansion for σ(E) in inverse powers of E, the coefficients of which are functions of the chemical binding of the host moderator atoms and its mass. This was a real advance and some modifications even allowed one to obtain the angular components $\sigma_n(E' \to E)$. But Corngold went one better; in a flash of inspiration he asked why be satisfied with just the cross section, why not consider the complete scattering term in the Boltzmann equation in the form

$$\frac{1}{2\pi m k} \int d\mathbf{k}_0 \phi(\mathbf{k}_0) \int_{-\infty}^{\infty} dt e^{it\Delta E} \langle G_{\beta\beta} \rangle$$

where $\phi(\mathbf{k}_0)$ is the flux of neutrons.

By changing to the energy and angle variables, and integrating out the angular variable, Corngold arrived at the following elegant expression

$$\int_0^\infty dE'\sigma(E'\to E)\phi(E') = \frac{\pi}{\mu}\sum_{n=0}^{\infty}\frac{(1-\mu)^n}{n!}\frac{1}{E^n}\left[\frac{\partial^n}{\partial y^n}\int_{[(\sqrt{y}-\mu)/(1-\mu)]^2 E}^{[(\sqrt{y}+\mu)/(1-\mu)]^2 E}\phi(E')\frac{dE'}{E'}S_n(\kappa^2)\right]_{y=1}$$

where $\mu = 1/A$

$$\kappa^2 = 2A\left(E' - \frac{y-\mu}{1-\mu}E\right)$$

and $S_n(\kappa^2)$ is the n$^{\text{th}}$ moment of the distribution in energy of scattered neutrons. S_n contains all the details of the state of chemical binding of the host moderator, its temperature and its mass.

What a beautiful result this is. For example, the term n=0 is

$$\frac{\sigma_f}{1-\alpha}\int_E^{E/\alpha}\frac{dE'}{E'}\phi(E')$$

i.e. the familiar slowing down result for scattering of neutrons against free nuclei at rest. Terms with higher values of n take into account the thermal motion and the chemical binding of the atoms containing the struck nuclei. Corngold then went on to use Mellin transforms to obtain a complete solution of the infinite medium transport equation in the form of a divergent, but very useful, series in inverse powers of E. He even discovered the existence of a new set of Placzek 'wiggles' arising from the higher order terms. After reading this paper in 1960, I was filled with a mixture of envy and admiration. There were two sequels to this classic paper. One which applied the theory to specific models by evaluating the $S_n(\kappa^2)$ terms for the isotropic harmonic oscillator, simple crystal lattices and for the bound proton in the water molecule. The latter enabled some experimental data on neutron spectra measured by Beyster (1959) to be compared with theory down to an energy of 0.2 eV. The agreement was excellent. A further paper with Larry Zamick in 1961 examined the effect of leakage. In 1963 I was also able to use Corngold's asymptotic series to treat the slowing down of a neutron pulse in the energy region E>0.2 eV, (Nuclear Sci. and Engng. **19** (1964) 221) to include the effects of thermal motion and chemical binding. I too found excellent agreement with some experimental results obtained by the University of Birmingham's Applied Nuclear Science Department. I also used the Corngold method to obtain spectra in the so-called 'Kottwitz problem' in which the distribution of neutrons across a temperature discontinuity was to be found. (Pulsed Neutron Research, IAEA, Vienna, **1** (1965) 607-621).

I cannot resist the temptation to include here some unpublished work which I did in 1967 and in which I extended Noel's method to deal with the Legendre components

$$\int_0^\infty dE'\sigma_l(E' \to E)\phi_l(E')$$

which arise when solving the spatially dependent transport equation. I will simply give the result without proof, viz:

$$\frac{\Sigma_f(1+\mu)^2}{4\mu}\sum_{n=0}^{\infty}\frac{(1-\mu)^n}{n!}\frac{1}{E^n}\left[\frac{\partial^n}{\partial y^n}\int_{\left[(\sqrt{y}-\mu)/(1-\mu)\right]^2 E}^{\left[(\sqrt{y}+\mu)/(1-\mu)\right]^2 E} P_l\big(\mu_0(E,E',y)\big)\phi_l(E')\frac{dE'}{E'}\widetilde{S}_n(\kappa^2)\right]$$

where

$$\mu_0 = \frac{E(y-\mu^2) - E'(1-\mu)^2}{2\mu(1-\mu)\sqrt{EE'}} \qquad and \qquad \widetilde{S}_n = S_n / S_0$$

This is a good place to note a disappointment: namely, that Noel has never written a book. A sad loss to the reactor physics community because not only is his knowledge and insight into reactor theory of the highest calibre, but he has a style of prose which could match that of Charles Dickens. Nowhere is this style better illustrated than in his review articles, e.g. "Quasi-exponential decay of neutron fields" (Advances in Nuclear Science and Technology **8** (1975) 1-46), and in a beautiful set of lecture notes on the scattering of thermal neutrons delivered to the Reactor Theory Division of Brookhaven National Laboratory in 1959. I never had the privilege of attending those lectures but they were reproduced as an internal Brookhaven report and encapsulate everything that was known about thermal neutron scattering up to that time. These lectures would have formed the basis for a book: Noel, why did you not publish them ?

2. RESONANCE CAPTURE AND LATTICE PHYSICS

I have been speaking about Noel's contributions to steady state, asymptotic thermal neutron spectra calculations, whereas he is probably best known for his work on the theoretical basis of the pulsed neutron experiment. Before discussing that work, however, the reader should also be reminded that Noel made some seminal contributions to the understanding of resonance escape probability and, as a consequence, to the theory of slab lattices. In a Brookhaven report, "Resonance Escape Probability in Slab Lattices" BNL-445 (T-93) published in 1956 and in condensed form in The Journal of Nuclear Energy, (**4**, (1957) 293), Noel developed a completely self-consistent theory of resonance absorption in lumped systems. Up to that time, the calculation of resonance absorption was for the most part semi-empirical and based on the early work of Wigner, who suggested that resonance absorption be divided into two parts: volume and surface terms. Thus one used the well-known recipe for the resonance integral in the form $I = a + b(S/M)$ where a and b depend on the fuel type. In later work the *S/M* term was modified to $(S/M)^{1/2}$, but the underlying theory was still intuitive as developed by Chernick (First Geneva conference, 1955) using collision probabilities. Corngold's contribution was to show that the collision probability method was the zeroth approximation of a solution of the Boltzmann equation when a space-angle expansion is made of the neutron flux in Legendre polynomials. In particular, Corngold found, for a repeating slab lattice, that higher order terms, which account for non-uniformity of the spatial flux, contribute little to the neutron absorption in those resonances which contribute most to the capture probability.

It is worth giving the essence of Corngold's technique because it had a significant influence on the solution of the Boltzmann equation for other energy dependent problems unconnected with resonance absorption. (e.g. thermal neutron fluxes in lattices, as in Henry Honeck's THERMOS code and Denis Newmarch's RIPPLE code). We start from a repeating slab lattice in which there are alternating layers of fuel and moderator. In the fuel, occupying the region $0 < x < a$, the flux ψ obeys the equation

$$\mu\frac{\partial\psi}{\partial x} + \Sigma_0\psi = T_0\psi + S$$

and in the moderator $a < x < a+b$, the flux ϕ obeys

$$\mu\frac{\partial\phi}{\partial x} + \Sigma_1\phi = T_1\phi$$

where T_i are the integral scattering operators, in the lethargy variable u, which describe the slowing down of neutrons in fuel and moderator (i=0,1), respectively. Using the reflective or specular boundary conditions, or in this case periodicity, it is possible to obtain coupled sets of integral equations for the angular and spatial moments of the fluxes, viz: $\psi_{ml}(u)$ and $\phi_{ml}(u)$, where

$$\psi(x,\mu,u) = \frac{2}{a}\sum_{n=0}^{\infty}\frac{2m+1}{2}\sum_{l=0}^{\infty}\frac{2l+1}{2}\psi_{ml}(u)P_m(\mu)P_l\left(\frac{2x-a}{a}\right)$$

with an analogous expansion for ϕ. The novel aspect here is a spatial expansion in Legendre polynomials as well as the conventional angular expansion. The resulting equations take the form

$$\psi_{sq}(u) = \sum_{lm} \alpha_l \alpha_m \Delta_{sl}^{qm} T_1^l \phi_{lm} + \frac{1}{2}\sum_m \alpha_m \Lambda_{s0}^{qm}(\Sigma_{s0}\psi_{0m} + S_{0m})$$

$$\phi_{sq}(u) = \sum_{lm} \alpha_l \alpha_m \overline{\Delta}_{sl}^{qm} T_1^l \phi_{lm} + \frac{1}{2}\sum_m \alpha_m \overline{\Lambda}_{s0}^{qm}(\Sigma_{s0}\psi_{0m} + S_{0m})$$

where $\alpha_l=(2l+1)/2$ and the Δ and Λ symbols are functions of the cross sections $\Sigma_0(u)$ and $\Sigma_1(u)$ and the geometrical factors a and b.

It is readily shown that the zeroth approximation to the above equations is

$$\psi_{00} = \frac{1}{4}\Delta_{00}^{00} T_1^0 \phi_{00} + \frac{1}{4}\Lambda_{00}^{00}(\Sigma_{s0}\psi_{00} + S_{00})$$

$$\phi_{00} = \frac{1}{4}\overline{\Delta}_{00}^{00} T_1^0 \phi_{00} + \frac{1}{4}\overline{\Lambda}_{00}^{00}(\Sigma_{s0}\psi_{00} + S_{00})$$

After some algebra, it can be shown that these equations are equivalent to the set obtained using the collision probability arguments of Chernick alluded to above. But, of course, by taking higher terms in the expansion, the errors in the simple theory can be assessed. Corngold presents a large amount of numerical work to test the accuracy of the zeroth order approximation. For example, using the eight lowest resonances in ^{238}U, he shows that inclusion of the next term in the expansion of the flux ψ changes the resonance integral by about 0.9%. This result is both a tribute to Jack Chernick's intuition and to Noel Corngold's powers of analysis.

I remember well as a graduate student working my way laboriously through Noel's elegant BNL report and feeling exceptionally pleased with myself as I was able to reproduce each equation and result. I learned a great deal about differential and integral forms of the transport equation during this period of study which stood me in good stead in the years to come.

This was not Noel's only contribution to resonance absorption, for in 1959 he published a paper entitled "Slowing Down of Neutrons in Infinite Homogeneous Media" in the Proceedings of the Physical Society (LXXA p 793). From heterogeneous to homogeneous systems, surely a retrograde step ? Not at all, since it is well known that most heterogeneous resonance absorption problems can be reduced to pseudo-homogeneous ones with effective parameters. The purpose of Noel's paper was to develop a variational method to solve the resonance absorption slowing down equation, or to be more precise the associated resonance integral. This had not been done before for a number of reasons, but in my view because most people associate variational methods with Fredholm integral equations whereas the slowing down equation is of the Volterra type. It was therefore necessary to do two things: (1) recast the slowing down equation into a form suitable for a variational attack, and (2) to develop a variational functional for the Volterra equation. Recasting was done by using a trick invented by George Placzek (Phys. Rev. **69** (1946) 423) who changed the conventional slowing down equation

$$\phi(u) = \int_0^u du' K(u-u')h(u')\phi(u') + \delta(u)$$

to the form

$$\phi(u) = h(0)\psi_m(u) - \int_0^u du' \psi_m(u-u')g(u')\phi(u')$$

where $\psi_m(u)$ is the Placzek function for zero absorption and can be obtained exactly. It exhibits the famous Placzek 'wiggles'. $g(u)=1-h(u)$ is the ratio of absorption to total cross section. In his original paper, Corngold generalises these equations to several species to account for mixtures of fuel and moderator. However, the method is more transparent if we forego that step (but essential for numerical results).

Resonance absorption occurs well below source energies and so $h(0)\,\psi_m(u)$ in the above equation may be replaced by ψ_{asy} where clearly (with $h(0)=1$)

$$\phi_{asy} = \psi_{asy}\left[1 - \int_0^\infty du g(u)\phi(u)\right]$$

For a unit source, $\psi_{asy} = 1/\xi$ and so

$$\phi_{asy} = \frac{1}{\xi}\left[1 - \int_0^\infty du g(u)\phi(u)\right]$$

But by definition, the resonance escape probability p is given by

$$p = 1 - \int_0^\infty du g(u)\phi(u)$$

and so the equation for ϕ may be rewritten

$$\phi(u) = \frac{1}{\xi(1-p)}\int_0^\infty du' g(u')\phi(u') - \int_0^u du' \psi_m(u-u')g(u')\phi(u')$$

To form a variational principle, it is necessary to define an adjoint function (unless the problem is self-adjoint), and this is clearly

$$\phi^+(u) = \frac{1}{\xi(1-p)}\int_0^\infty du' g(u')\phi^+(u') - \int_u^\infty du' \psi_m(u'-u)g(u')\phi^+(u')$$

The trick when finding the adjoint of a Volterra operator is to examine the domain of integration carefully and hence the limits (u,∞). I found this in Morse and Feshbach (1953, p877), Noel where did you find it ?

A variational principle is then defined by Corngold in terms of ϕ and ϕ^+ and whose stationary value is

$$J_{st} = \frac{1}{\xi(1-p)}$$

from which p can be calculated. Corngold illustrated his work by applying it to two resonances in ^{238}U which are neither 'narrow' nor 'wide', viz: the ones at 36.9 eV and 192 eV. He obtained the following results

resonance	1-p(exact)	1-p(variational)	1-p(narrow)	1-p(wide)
36.9	0.05820	0.05682	0.04739	0.06110
192	0.007119	0.007630	0.005068	0.01228

Confirmation indeed of the power of the variational method. I have to admit that I took over Noel's method, 'lock, stock and functional' and applied it to the slowing down of 'hot atoms' in a host gas and obtained excellent results (Williams,1976, unpublished but I don't know why !)

3. PULSED NEUTRONS AND A VISIT TO THE NEW WORLD

I now pass on to the work for which Noel is rightly acknowledged the master. Namely, the theoretical basis of the decay of neutron pulses in moderators. Pulsed neutron research had become an interesting topic for theoreticians since its inception in the early 1950's by von Dardel (1953) in Sweden and his discovery of 'diffusion cooling'. In the late 50's and early 60's, as the experimental work burgeoned, it was noticed by a number of experimentalists that, contrary to the conventional wisdom, the asymptotic decay of a neutron pulse in a finite block of moderator, was not exponential. Rather, it was *sometimes* necessary to fit the decay curve by a set of exponentials or even by a function of the form $t^a \exp(-bt)$. Such behaviour was difficult to understand.

In a pioneering work, Corngold, with his co-workers Paul Michael and Warren Wollman (Nucl. Sci. Engng. **15** (1963) 13-19), first examined the mathematical structure of the infinite medium, time-dependent equation for neutron slowing down in the form

$$[x\Sigma_s(x) - \lambda_m]N_m(x) = \int_0^\infty dx' x' \Sigma_s(x' \to x) N_m(x')$$

where the velocity is x and the time dependent flux N(x,t) is written

$$N(x,t) = \sum_m C_m N_m(x) e^{-\lambda_m t}$$

Up to this time, it had generally been assumed that the scattering operators representing physically reasonable models of moderating materials would possess an infinite number of discrete eigenvalues extending from zero to infinity, with no limit point other than that at infinity. Associated with these eigenvalues would be eigenfunctions which form a complete set. In the paper cited above, Corngold and his co-workers showed that, for the case of the proton gas, (i.e. gas model with A=1), this was certainly not true and that the eigenvalue spectrum consisted of an infinite number of discrete points in the range $0 \leq \lambda < (v\Sigma)_{\min}$,with $(v\Sigma)_{\min}$ being a limit point. They also found that a continuous spectrum exists for $\lambda > (v\Sigma)_{\min}$. This quantity, $(v\Sigma)_{\min}$, assumed considerable importance in the 1960's and became a sort of 'holy grail', both revered and reviled by theoretician and experimentalist alike. Because of the nature of the total cross section and its behaviour with velocity for all common moderators, the value of $(v\Sigma)_{\min}$ occurs as $v \to 0$. Indeed, using the scattering theory developed in terms of the collision time t, it is possible to write down a general expression for $(v\Sigma)_{\min}$ in the form

$$(v\Sigma)_{\min} = \frac{\Sigma_b}{\sqrt{8\pi}} \int_{-\infty}^{\infty} \frac{dt}{\left[W(t)/A - it\right]^{3/2}}$$

where $W(t)$ contains the chemical binding properties and the temperature of the moderator atoms. For example for an ideal gas

$$(v\Sigma)_{\min} = \left(\frac{8kT}{\pi M}\right)^{1/2} \Sigma_f$$

whereas for a crystalline moderator,

$$(v\Sigma)_{\min} = \frac{v_0 \Sigma_b}{A} \int_0^{\infty} \frac{\sqrt{w}\, f(w)\, dw}{e^{w/T} - 1}$$

$f(w)$ being the phonon frequency distribution of the crystal lattice.

This early work of Corngold was important in that it showed the limitations of the simple theories used at that time. It also pointed the way towards an interpretation of the experimental results in view of the presence of the non-exponential component of flux arising from the continuous spectrum.

Corngold, in collaboration with Ivan Kuscer, (a physicist of great talent and charm), extended the work on the existence of eigenvalues in infinite media to the case where the moderator was a gas of atomic mass $A \geq 1$, a liquid or a solid (Phys. Rev. **139** (1965) A981). They were able to confirm the existence of an infinite number of eigenvalues in the range $0 \leq \lambda < (v\Sigma)_{\min}$ for gases of all masses and to find the rate at which these eigenvalues approached the limit point. Even more striking were the conclusions on solids, where it was shown that only a finite number of eigenvalues existed, or none at all, in the range $[0, (v\Sigma)_{\min}]$. The existence of a continuous spectrum from $(v\Sigma)_{\min}$ to infinity was also demonstrated. This was very exciting and was further evidence of the information hidden away in the transport equation. It certainly sent important signals to experimentalists.

In the last paragraph, I jumped ahead to 1965 to complete the picture on infinite media. But now I must backtrack to 1962 when I arrived at Brookhaven National Laboratory. However, before discussing the exciting work being carried on there under Noel's direction, I would like to take the liberty of telling the reader about my first encounter with Noel. As I intimated in my introduction above, I had been in correspondence with Noel when I was a graduate student and he very patiently answered some queries that I had on synthetic differential kernels (all the rage in those days, alas now made redundant by 'gigabytes'). On completion of my doctorate, I submitted a paper to the Brookhaven Conference on Neutron Thermalisation held in 1962 and, on the basis of that (I think), Noel suggested that I apply for a Research Associateship at Brookhaven. This I did, and to my surprise and everlasting gratitude, I was successful.

My wife and I travelled to America in August 1962 on the Holland-America line's SS Maasdam. The journey took 10 days, we hit a hurricane and I was terribly seasick. I also broke a basin in the cabin. Imagine my feelings, therefore, when only two days ago (i.e. August 1998), I received a letter written on Holland-America line headed notepaper. 'Blimey', I thought, Holland-America line have caught up with me at last and want me to pay for the broken basin. But no, it was a letter from Noel telling me about a cruise that he and Cynthia had taken to Alaska. I understand the ship was rather larger than the Maasdam !

Turning now to my arrival in the 'New World'. We sailed into New York Harbour and up the Hudson River at about 4.00am and for the first time I saw the lights of the City. It was a magical moment. I still have the letter which Noel sent me offering assistance on my arrival. It is a treasured possession and I have reproduced it below on the next page.

As it happens, we had decided to travel direct to Brookhaven and were met at Hoboken, New Jersey, where the ship eventually docked, and not the best place to visit first, by a very friendly Brookhaven driver and one of the largest cars I had ever seen. From Hoboken, we were driven up Long Island to Brookhaven which is close to the hamlet of Yaphank. The culture shock in coming from a Britain that had no motorways and was still recovering from the second world war, to a USA which was at the height of its prosperity (remember, this was the 'Happy Days' era), was overwhelming. On arrival at Brookhaven, we were taken to a so-called efficiency apartment which was palatial compared with the spartan 100 year old flat we had occupied in South London. We arrived on a Saturday and had a very exciting weekend. Noel phoned on the Sunday evening and said he would collect me the next day. When the knock on the door came, I could hardly believe my eyes. I was expecting some aging, grey-haired scientist but instead there was this young man. Actually he was about 7 years older than me but looked about the same age. Over the years he has changed little as our cover picture shows. That was the beginning of a valued friendship and source of scientific advice that I always recall with gratitude.

BROOKHAVEN NATIONAL LABORATORY
ASSOCIATED UNIVERSITIES, INC.
UPTON, L.I., N.Y.
TEL. YAPHANK 4-6262

REFER:

August 20, 1962

Dear Dr. Williams:

Your apartment at Brookhaven will be ready for you on 7 September, the date of your arrival. Do you intend to travel directly to the Laboratory on the 7th? or do you intend to spend the week-end in Manhattan? If the former, we will send an automobile and driver to meet you and help you with your luggage. If the latter, we can make a hotel reservation for you in New York, and you can proceed to the Laboratory by train on Monday, the 10th. Which do you prefer?

We look forward to meeting you.

Sincerely yours,
Noel Corngold

Perhaps one more anecdote. During the summer of 1963, Noel had to go to a meeting out of town. Before going he said to me: 'Mike, we have a consultant visiting the Group tomorrow and as I will not be available, will you speak to him about the Group's activities'. 'Glad to help I said, what is his name ?' 'Garrett Birkhoff' said Noel laconically. Fortunately, the great man was charming, treated me as if I were his equal, and I learnt much.

It is time now to return to technical matters and, in particular, the pulsed neutron work that was being carried out by the Reactor Theory Group in Building T-135 (Now alas no more). At that time, Noel had a graduate student by the name of Charles Shapiro (Chuck in those days) who was working on the use of the degenerate kernel method for calculating the time decay constants in finite systems characterised by a buckling. The infinite medium is of course a special case with zero buckling. Afficionados will know what buckling means and so I will not elaborate. However, it should be pointed out that it is only an approximate way of dealing with finite media and there are more accurate methods. Nevertheless, buckling is a powerful concept and Noel and Chuck were working on methods to show how the time decay constants changed as the buckling was increased from zero. The work was eventually published in the Physical Review (Phys. Rev. **137** (1965) A1686) but right now I am looking at Shapiro's Ph.D thesis, "Time Eigenvalues and Degenerate Kernels in Neutron Thermalization", which has been in my library for nearly 35 years. The basis of the method was this. Consider the Boltzmann equation in the form

$$\left[\frac{\partial}{\partial t} + v\left(\Sigma_s(v) + \Sigma_a(v)\right) + v\Omega.\nabla\right] N(\mathbf{r}, v, \Omega, t)$$

$$= \int_0^\infty dv' v' \int d\Omega' \Sigma_s(v' \to v; \Omega'.\Omega) N(\mathbf{r}, v', \Omega', t)$$

Assume isotropic scattering and consider the elementary solution

$N(\mathbf{r}, v, \Omega, t) = n(\lambda, B, v, \Omega) e^{-(\lambda+\lambda_0)t + i\mathbf{B}.\mathbf{r}}$

The transport equation now becomes, with $v\Sigma_a(v)=\lambda_0$,

$$\left[-\lambda + v\Sigma_s(v) + iv\mathbf{B}.\Omega\right] n(\lambda, B, v, \Omega) = \frac{1}{2}\int_0^\infty dv' v' \Sigma_{s0}(v' \to v) n_0(\lambda, B, v')$$

Dividing by the quantity in square brackets and integrating over the angle between $\mathbf{B}$ and Ω, we find

$$n_0(\lambda, B, v) = \frac{1}{vB}\tan^{-1}\left[\frac{vB}{v\Sigma_s(v) - \lambda}\right] \int_0^\infty dv' v' \Sigma_{s0}\left(v' \to v\right) n_0(\lambda, B, v')$$

Corngold and Shapiro now represent the scattering function by a degenerate kernel as follows

$$v'\Sigma_{s0}(v' \to v) = M(v)\sum_{i,j}^{K} a_{ij}\Sigma_i(v')\Sigma_j(v)$$

where a_{ij} and $\Sigma_i(v)$ can be put in terms of moments of $\Sigma_{s0}(v' \to v)$. As far as I know, this was the first time that the scattering kernel had been represented in degenerate form, which is surprising because in many books on integral equations it is quoted as a standard method of approach. However, the clever part of the Corngold-Shapiro representation is that there are only a finite number of terms in the sum and, as the number of terms is increased, the earlier terms are not affected. Thus one can successfully preserve the total cross section (vital for $(v\Sigma(v))_{\min}$ calculations), the first velocity moment and so on. This allows more and more scattering law information to be inserted into the Boltzmann equation and is far superior to expanding the flux in energy polynomials which hitherto was the preferred method of approach.

Inserting the degenerate kernel expansion into the equation for n_0, multiplying by $\Sigma_k(v)$ on both sides and integrating over velocity leads to a set of homogeneous algebraic equations for the quantities

$$\tilde{n}_j = \int_0^\infty dv\Sigma_j(v)n_0(\lambda, B, v)$$

The determinant of the coefficients then leads to a transcendental equation for the eigenvalues $\lambda_n(B^2)$. The major conclusion of this work was that, regardless of the scattering model, there exists a maximum value of the buckling beyond which there are no discrete eigenvalues. This is a conclusion of the utmost importance not least for the interpretation of experimental results. In fact, numerical results show that for crystalline moderators (Wood, 1965, PhD thesis, University of Birmingam,UK) the value of $B^2_{\max}$ for the Debye model with a Debye temperature of 3.4T, simulating beryllium, is equal to 0.039 cm^{-2} corresponding to a cube of side 27.6 cm. Such a value is well above some moderator sizes used in experiments and explains the anomalous decay observed. It is difficult to know whether the experimentalists of the day, of whom there were many, appreciated these subtleties. Certainly some did and one should note the very nice work of the Australians at Lucas Heights under the direction of Ian Ritchie (J. Nuclear Energy, **22** (1968) 371,735). The work of Andrews at UCRL (UCRL-6083, 1960) was also of very high precision and proved useful for measuring deviations from conventional theory, as did the work of de Saussure and Silver (BNL conference **4** (1962) 981). The later work of one of Noel's students, Harold McFarlane, should also be noted (Nucl. Sci. Engng. **49** (1972) 438).

The work carried out by Shapiro and Corngold described above was numerical and could not therefore be described as a proof of the maximum buckling effect. Once more, therefore, we see Corngold in action doing what he does so elegantly. Namely, a proof that a maximum buckling exists by considering the existence of solutions of the Boltzmann equation. This was done for a general scattering kernel with only the obvious physical limitations on its behaviour. This proof can be found in Nuclear Science and Engineering **19** (1964) 80-90 with a companion paper, co-authored with Paul Michael, in which the implications for experiment are pointed out. This paper very clearly shows how at least 50% of the fundamental decay constants measured in beryllium at that time had exceeded the value of $(v\Sigma)_{min}$ for this element, viz: $4\ 10^{-3}\ s^{-1}$. The critical buckling value found was just around $0.03\ cm^{-2}$ and in reasonable agreement with Wood's value noted above who treated beryllium as a Debye crystal.

To be historically correct, the influence of the size of the moderator on the existence of the discrete time eigenvalues was independently predicted by Mark Nelkin (Physica, **29** (1963) 261) who used the integral equation and a separable kernel for energy exchange. He came to the same conclusion as Corngold and thereby confirmed that the effect was not some artefact of the buckling approximation.

4. SPATIAL RELAXATION

Also present in the paper which proved the maximum buckling theorem, was a proof of the maximum absorption theorem in the diffusion length experiment. This considers a half-space of absorbing moderator with an absorption cross section that goes as 1/v as $v\to 0$. If a beam of neutrons is incident on the face of a half-space at z=0, then normally one would expect after the surface transients have died away, the flux to obey the exponential law, i.e.

$$n_0(z,E) \approx A(E)e^{-\kappa z}$$

where κ is the spatial eigenvalue and the diffusion length $L=1/\kappa$. Corngold showed that if the absorption cross section is written as $\Sigma_a(v) = \alpha\sigma_a(v)$, then there exists a maximum value of α , say α^* , such that for $\alpha > \alpha^*$ no discrete eigenvalues, κ, exist in the range $0 \le \kappa < \Sigma_{min}$ for the equation

$$\tilde{n}_0(v) = \frac{1}{2\kappa v} Log\left[\frac{\Sigma(v)+\kappa}{\Sigma(v)-\kappa}\right]\int_0^\infty dv' v' \Sigma_{s0}(v' \to v)\tilde{n}_0(v')$$

Note that in the spatial relaxation problem, Σ_{min} , i.e. the minimum value of the total cross section, replaces $(v\Sigma)_{min}$ in the pulsed neutron problem. Note also that whilst $(v\Sigma)_{min}$ occurs as $v \to 0$, Σ_{min} can occur at various energies according to the nature of the moderator. For example, in gases or liquids, the scattering cross section generally decreases smoothly from infinity at $v=0$ to the free atom value at around 1 eV. Thus Σ_{min} in this case will be the free atom value as used in slowing down theory. However, in crystalline materials, which exhibit interference effects, the minimum value will lie at the Bragg cut-off energy which is often in the energy region of 0.001 eV. For example, in bismuth, $1/\Sigma_{min}$ =65 cm and since the measured diffusion length in bismuth is less than this, one may conclude that no truly asymptotic mode can exist in this element.

It was the results that Noel obtained for the diffusion length experiment together with some comments by Glen Price (of BNL), that led me to inquire into the effect of buckling on the diffusion length. In other words, in addition to absorption, how did transverse leakage in a finite block of moderator affect the fundamental spatial relaxation mode ? My studies led to a theorem for the exponential experiment analogous to Corngold's for the pulsed neutron experiment. Namely, that there existed a maximum value for the transverse buckling in a diffusion length experiment in excess of which no discrete eigenvalue of the transport equation existed (IAEA conference on neutron thermalisation and reactor spectra, Ann Arbor, 1968, vol I, p27). It turned out that, for graphite and beryllium, the transverse dimensions corresponding to these maximum bucklings were 115 and 140 cm, respectively. Since many diffusion length experiments had been performed on systems whose dimensions were far less than these values, the theorem placed some doubt on the measured values of the diffusion lengths calculated by a number of workers. In a series of experiments designed to detect the non-exponential decay in graphite, de Juren and Swanson (J. Nuclear Energy, **20** (1966) 905) found deviations at approximately the bucklings predicted. Corngold's work on eigenvalue existence therefore had a profound influence on workers in related fields.

Corngold's work on the maximum absorption theorem was developed by his student Jack Dorning (Dorning and Thurber, Trans. Am. Nucl. Soc. **11** (1968) 579) who showed that a pseudo- relaxation length could exist in the form of a pole embedded in the continuum or, alternatively, by a continuum eigenfunction which had a very sharp peak at a particular value, thereby simulating a delta function. These methods were to some extent already known in rarefied gas theory where they were used to study the dispersion law of sound propagation.

The use of the buckling approximation has also been employed by James Duderstadt, another of Corngold's students, to explain various phenomena regarding neutron wave propagation in crystalline moderators. This work showed the existence of a greater richness in the continuous spectrum, which exhibited disjoint regions as well as 'whiskers' corresponding to Bragg cut-off effects. It came as no surprise, although the result was pretty, to find that as the frequency, ω, of the neutron wave increased, there existed a maximum value ω^* beyond which no wave of the form $e^{i\omega t-\kappa(\omega)z}$ would propagate. Beyond ω^* the motion could only be described by continuum eigenfunctions with the curious 'whiskers', leading to pseudo-discrete eigenvalues embedded in the continuum.

Many workers benefited from the seminal work of Corngold, and his research students had a great start in life by being associated with such a scholar. But I am sure they know that.

5. THERE IS NO EPILOGUE

The wave propagation work takes the Corngold story to around 1975 and an excellent summary is to be found in two of Noel's review articles: "Some recent results in the theory of thermal neutrons", SIAM-ANS Proceedings, vol I (1969) pp79-91 and "Quasi-exponential decay of neutron fields", cited earlier. From the point of view of this appreciation, it is also a good point to stop. For although Noel has made many more contributions to transport theory, it would take equally more pages to do them justice. Also, those early years were the ones in which I lived and breathed transport theory and was fully aware of what was being done. After that time, my interests moved to reactor noise, rarefied gas dynamics and radiation damage and I therefore feel less confident about describing the overall picture of transport theory. It is also true to say that reactor physics became more 'computerised' in the 1970's and, at least for me, less interesting. There were areas of interest such as neutron streaming in lattices which required cunning analysis, but the glory days were over (or was it old age descending ?). Certain aspects of radiation damage theory and fragmentation which were describable by a transport equation continued to demand old-fashioned theory and, in the only joint paper I have co-authored with Noel, we described some anomalous results arising from the use of a particular scattering law, which appeared to violate conservation of energy and mass (Operator Theory: Advances and Applications, **51** (1991) 89). The reader will be relieved to learn that we located the source of the problem.

I have attempted in this appreciation to my friend and colleague of some 35 years to give a personal view of his contributions, not just to reactor theory but also on how they have affected me and many other people who have worked with him or under his guidance. I have always felt that anyone who gained a PhD under Noel's supervision should have some special designation, e.g. PhD(NDRC). For me, my time at Brookhaven was the most important period in my professional life. It enabled me to meet all the current icons of reactor physics, it established the pattern of my research for years to come and it gave me confidence to write my first book "The Slowing Down and Thermalization of Neutrons", which I dedicated to the Yaphank Philosophical Society. It could equally well have been dedicated to Noel.